# Rotational and vibrational analysis of the $\mathrm{CaF} \boldsymbol{B}^{2} \Sigma^{+}-X^{2} \Sigma^{+}$system 

Michael Dulick, Peter F. Bernath, and Robert W. Field<br>Spectroscopy Laboratory and the Department of Chemistry, Massachusetts Institute of Technology, Cambridge, MA 02139, U.S.A.

Received December 17, 1979


#### Abstract

A rotation-vibration analysis of the $\mathrm{CaF} B^{2} \Sigma^{+}-X^{2} \Sigma^{+}$system is reported. Excitation spectra of CaF are recorded with a $\mathrm{cw}, 1 \mathrm{MHz}$ bandwidth. dye laser combined with restricted-bandpass (2 $\AA$ ) fluorescence detection. The accuracy of line measurements is $0.003 \mathrm{~cm}^{-1}$. Nine bands in the $\Delta v=0$ sequence are analyzed ( $v^{\prime \prime}=0-2,8,9,12-15$ ) and selected lines in four $\Delta v=-1$ bands ( $v^{\prime \prime}=$ $1-4$ ) are used to obtain band origins needed for the vibrational analysis. The main constants ( $1 \sigma$ error in parentheses) for the CaF $B^{2} \Sigma^{+}$state are: $T_{\mathrm{e}}=18841.309(3) \mathrm{cm}^{-1}, B_{8}=0.342604(7) \mathrm{cm}^{-1}$, $\omega_{\mathrm{e}}=572.405(36) \mathrm{cm}^{-1}, \alpha_{\mathrm{e}}=0.002630(6) \mathrm{cm}^{-1}, \omega_{\mathrm{e}} x_{\mathrm{e}}=3.143(13) \mathrm{cm}^{-1}, R_{\mathrm{e}}=1.955 \AA$. The $B^{2} \Sigma^{+}$ state is found to be in pure precession with $A^{2} \Pi$ with $/=1$.


#### Abstract

On présente une analyse rotation-vibration du système $B^{2} \Sigma^{+}-X^{2} \Sigma^{+}$de CaF . Les spectres d'excitation de CaF ont été obtenus en utilisant un laser à colorant, à onde continue et de 1 MHz de largeur de bande, en combinaison avec un détecteur de fluorescence à largeur de bande limitée ( $2 \AA$ ). La précision de la mesure des raies est de $0.003 \mathrm{~cm}^{-1}$. Neuf bandes dans la série $\Delta v=0$ sont analysées $\left(v^{\prime \prime}=0-2,8,9,12-15\right)$; et des raies choisies dans quatre bandes $\Delta v=-1\left(v^{\prime \prime}=1-4\right)$ sont utilisées pour obtenir les origines des bandes requises pour l'analyse vibrationnelle. Les principales constantes de l'état $B^{2} \Sigma^{+}$sont (avec la valeur de l'écart $\sigma$ entre parenthèses) : $T_{e}=$ $18841.309(3) \mathrm{cm}^{-1}, B_{\mathrm{e}}=0.342604(7) \mathrm{cm}^{-1}, \omega_{\mathrm{e}}=572.405(36) \mathrm{cm}^{-1}, \alpha_{\mathrm{e}}=0.002630(6) \mathrm{cm}^{-1}, \omega_{\mathrm{e}} x_{\mathrm{e}}=$ $3.143(13) \mathrm{cm}^{-1}, R_{\mathrm{c}}=1.955 \AA$. On trouve que l'état $B^{2} \Sigma^{+}$est en précession pure avec $A^{2} \Pi$ pour $l=1$.


[Traduit par le journal]
Can. J. Phys., 58, 703 (1980)

## Introduction

The first complete vibrational analyses of the $\mathrm{CaF} X^{2} \Sigma^{+}, A^{2} \Pi$, and $B^{2} \Sigma^{+}$states were reported by Johnson (1) in 1929 followed by Harvey (2) in 1931. It was not until the advent of the tunable single mode dye laser, a lapse of over 40 years, that a correct rotational analysis of the $A^{2} \Pi-X^{2} \Sigma^{+}(0,0)$, $(0,1)$, and ( 1,0 ) bands appeared in 1975 (3). Recent Microwave-Optical Double Resonance (MODR) experiments on the $A-X$ system by Nakagawa et al. (4) and Optical-Optical Double Resonance (OODR) measurements involving the $X, A, E$, and $E^{\prime}$ states by Bernath and Field (5) improved the accuracy of the $A$ and $X$ state constants.

We present a complete rotational and vibrational analysis of the $B^{2} \Sigma^{+}-X^{2} \Sigma^{+}$system for selected bands of the $\Delta v=0$ and -1 sequences up to $v^{\prime}=15$ and $v^{\prime \prime}=15$. Previous analyses of the system by Mohanty and Upadhya (6), Khanna and Dubey (7), and Nanda and Mohanty (8) are shown to be incorrect.

The $B-X$ system spans the wavelength region $5145-5500 \AA$ and consists of closely spaced bands comprising the $\Delta v=0,1$, and -1 sequences. Because the internuclear distances and potential curves are nearly identical for the two states, one finds only an intense $\Delta v=0$ sequence and weaker $\Delta v= \pm 1$ sequences. The $\Delta v=0$ and +1 bands
display double heads shaded to the red (2) while the $\Delta v=-1$ bands appear to be headless. The double heads can be explained by the large spin-rotation constant of the $B^{2} \Sigma^{+}$state, which has been determined from our analysis to be in pure precession with the $A^{2} \Pi$ state. The result is complete separation of the four main branches $\left(P_{1}, P_{2}, R_{1}\right.$, and $R_{2}$ as expected for a ${ }^{2} \Sigma-^{2} \Sigma$ transition) of each band. Nearly identical rotational constants for the $X$ and $B$ states result in bandhead to origin separations as large as $15 \AA$ for the $(0,0)$ band compared to $6 \AA$ separations of successive bandheads in the $\Delta v=0$ sequence. The large head-origin separations, combined with the small spacing of heads within a sequence, result in extensive band overlap which renders an analysis using classical spectroscopic techniques a formidable task. The main problem is to separate lines belonging to a single vibrational band and to correctly identify and number the rotational branches belonging to a particular band of a sequence. As an example, we observe for $v>0$ in the $\Delta v=0$ sequence typically 12 strong lines per reciprocal centimetre with Doppler widths (FWHM) on the order of $0.023 \mathrm{~cm}^{-1}$. Usually strong lines from at least three different members of the $\Delta v=0$ sequence can be found in $1 \mathrm{~cm}^{-1}$ of the spectrum.

The method we used in our experiments involves
excitation of the CaF $B-X$ system by scanning a cw , narrow band ( $<1 \mathrm{MHz}$ ), single-mode dye laser over consecutive $1 \mathrm{~cm}^{-1}$ intervals and detecting the resultant fluorescence through a moderate resolution monochromator functioning as a narrow bandpass filter (typically $2 \AA$ spectral width). The method described is similar to that employed by Demtröder (9) and Linton (10) used in the analyses of $\mathrm{NO}_{2}$ and YO spectra, respectively. Two important advantages of narrow band detection vs. total fluorescence detection are: (I) elimination of detected scattered laser light and chemiluminescence which results in enhanced signal-to-noise ratio in the excitation spectrum, and (2) in the case of the CaF $B-X$ system, the excitation spectrum of the $\Delta v$ band of interest is simplified by exclusion of other bands simultaneously excited by the laser.

## Experimental

CaF was produced in a Broida-type oven (11) by the reaction of $\mathrm{SF}_{6}$ with calcium metal vapor produced by heating calcium metal in an alumina crucible and entraining the vapor in flowing argon at pressures typically $1-2$ Torr. The $\mathrm{Ca}-\mathrm{SF}_{6}$ reaction is very exothermic with an estimated enthalpy of $-36.5 \mathrm{kcal} / \mathrm{mol}(12,13)$. As a result, high vibrational levels of the $X$ state were significantly populated. This allowed us to observe, in a broad band dye laser ( $1 \AA$ FWHM bandwidth) excitation spectrum, bandheads from the $\Delta v=0$ bands as high as $v^{\prime \prime}=25$. Insufficient single line fluorescence intensity limited single-mode dye laser excitation to levels not much higher than $v^{\prime \prime}=15$ in the $\Delta v=0$ sequence.
$B-X$ fluorescence was excited by a Coherent Radiation Model 599-21 dye laser $(<1 \mathrm{MHz}$ bandwidth) operated with rhodamine 110 dye pumped by 4 W all lines from a CR-3 argon ion laser. It was necessary to adjust the pH of the dye solution, which is normally pH 6 , up to pH 7 in order to achieve single mode operation in the wavelength region $5296-5380 \AA$. The pH of the dye solution was increased by the addition of a commercial pH 10 buffer solution containing a mixture of potassium hydroxide and potassium carbonate in water. The buffer solution was slowly added to the circulating glycol-dye solution until a final single mode output power of 20 mW from the dye laser at $5296 \AA$ was achieved. The resultant $\mathrm{CaF} B-X$ fluorescence is dispersed through a Spex Model 1802 monochromator and detected by a dry ice cooled Hamamatsu R818 photomultiplier tube operated at 600 V .

Excitation spectra for the $\Delta v=0$ sequence were obtained selectively, two branches of a given
vibrational band at a time. The single mode dye laser was scanned through the selected branches (e.g., $R_{1}$ and $R_{2}$ ) in $1 \mathrm{~cm}^{-1}$ intervals and fluorescence was detected exclusively in the other main branches (e.g., $P_{1}$ and $P_{2}$ ) which arise from the desired $v^{\prime}$ and restricted range of $N^{\prime}, J^{\prime}$ levels. The monochromator wavelength setting and the spectral band width were adjusted to eliminate extraneous lines from other ( $v^{\prime}, v^{\prime \prime}$ ) transitions and yet allow a large enough bandwidth to observe a sufficient number of rotational lines over several $1 \mathrm{~cm}^{-1}$ scans without the need for frequent readjustment of the wavelength setting. For $\Delta v=-1$ sequence excitation scans, the monochromator was set to detect fluorescence in a rotational branch of the corresponding $\Delta v=0$ sequence band which is much stronger than that in $\Delta v= \pm 1$ bands. The fluorescence excitation spectrum is calibrated with respect to $\mathrm{I}_{2} B 0_{\mathrm{u}}^{+}-X^{1} \Sigma_{\mathrm{g}}{ }^{+}$lines (14) by simultaneously recording $I_{2}$ and CaF excitation spectra along with etalon fringes from a 750 MHz free spectral range semi-confocal Fabry-Perot. This allows the line positions to be measured to a relative precision of $\pm 0.002 \mathrm{~cm}^{-1}$ and an absolute accuracy of $\pm 0.003 \mathrm{~cm}^{-1}$. The absolute wave numbers of the line positions and band origins listed here have not been corrected by subtraction of $0.0056 \mathrm{~cm}^{-1}$ as suggested by Gerstenkorn and Luc (15).

## Results

A list of the measured line positions fitted to obtain molecular constants from the $\Delta v=0$ and -1 bands appears in the Appendix. It should be noted that the lines indicated as blended, in most cases, involve accidental overlap between similar $N$ members of different $F_{1}$ and $F_{2}$ branches (e.g., $P_{1}$ and $P_{2}$ ) both belonging to the same vibrational band. An ordinary absorption or emission spectrum would exhibit far more blends than our selectively detected fluorescence excitation spectra. The few exceptions to this blend-only-within-a-vi-brational-band rule involve the $(0,0),(1,1),(1,2)$, and $(2,3)$ bands. Laser excitation in the $(0,0)$ and $(1,1)$ bands produces extremely bright fluorescence. The monochromator spectral bandwidth was adjusted to $2 \AA$ for the $(1,1)$ band. At this setting, a small amount of leakage of $(0,0)$ fluorescence was unavoidable. This posed no problem in the analysis of the excitation spectrum of the $(1,1)$ band.

## I. Rotational Analysis

The absolute numbering of the rotational branches was established by a procedure involving combination differences between the $F_{1}$ and $F_{2}$

Table 1. $X^{2} \Sigma^{+}$rotational constants in reciprocal centimetres

| $v^{\prime \prime}$ | $B_{v}{ }^{\prime \prime}$ |  | $\mathcal{D}_{v}{ }^{\prime \prime} \times 10^{7}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Experimental* | Exp. - cal. | Experimental* | Exp. - cal. |
| 0 | 0.342488 | $-0.000002$ | $4.50 \dagger$ | 0.07 |
| 1 | 0.340050 | 0.000001 | $4.50 \dagger$ | 0.02 |
| 2 | 0.337621 (16) | 0.000000 | $4.50+$ | $-0.02$ |
| 8 | $0.323300(17)$ | -0.000014 | 4.73(4) | -0.06 |
| 9 | $0.320969(6)$ | $-0.000005$ | 4.73! | -0.10 |
| 12 | $0.313929(53)$ | 0.000097 | 5.04 § | 0.08 |
| 13 | $0.311781(13)$ | 0.000045 | 5.04(4) | 0.03 |
| 14 | $0.309413(35)$ | -0.000044 | 5.04 § | -0.01 |
| 15 | $0.307128(24)$ | -0.000063 | 5.04 § | $-0.05$ |

*Uncertainties in parentheses represent one standard deviation.
$\dagger$ Fixed to $D_{0}{ }^{\prime \prime}$, in ref. 5.
Fixed to $D_{8}^{\prime \prime \prime}$ in fit of line positions.
$\$$ Fixed to $D_{13}$ " in fit of line positions.
Table 2. $B^{2} \Sigma^{+}$rotational constants in reciprocal centimetres

| $v^{\prime \prime}$ | $B_{v}{ }^{\prime}$ |  | $D_{v}{ }^{\prime} \times 10^{7}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Experimental* | Exp. - cal. | Experimental* | Exp. - cal. |
| 0 | $0.341285(3)$ | $-0.000006$ | 4.65(2) | $-0.03$ |
| 1 | $0.338677(2)$ | 0.000003 | 4.75 (2) | 0.03 |
| 2 | $0.336074(17)$ | 0.000005 | $4.75 \dagger$ | -0.01 |
| 8 | $0.320691(15)$ | $-0.000014$ | 5.01 (3) | 0.02 |
| 9 | $0.318172(8)$ | $-0.000016$ | 4.95 (3) | $-0.08$ |
| 12 | $0.310594(57)$ | $-0.000118$ | 5.25\% | 0.10 |
| 13 | $0.308284(12)$ | 0.000039 | 5.25(4) | 0.06 |
| 14 | $0.305750(31)$ | $-0.000041$ | 5.25 + | 0.02 |
| 15 | 0.303311 (30) | -0.000038 | $5.25{ }^{+}$ | -0.02 |

* Uncertainties in parentheses represent one standard deviation.
+ Fixed to $D_{1}$ ' in fit of line positions.
component lines of a $\Delta N=+1$ or -1 branch. The first combination difference between a $P_{1}(N)$ and $P_{2}(N)$ line is (ref. 16, p. 249)
[1] $\Delta v_{12}{ }^{P}(N)=\Delta \gamma N-\frac{1}{2}\left(\gamma^{\prime}+\gamma^{\prime \prime}\right)$
while, between $R_{1}(N)$ and $R_{2}(N)$ lines, it is
[2] $\quad \Delta \nu_{12}{ }^{R(N)}=\Delta \gamma N+\frac{1}{2}\left(3 \gamma^{\prime}-\gamma^{\prime \prime}\right)$
In both cases the second combination difference is equal to $\Delta \gamma$, the difference between the upper and lower state spin-rotation constant. For the $B-X$ transition of $\mathrm{CaF},\left|\gamma^{\prime}\right| \gg\left|\gamma^{\prime \prime}\right|$ and to a first approximation the second difference equals $\gamma^{\prime}$. For the $A$ state $p<0$ (refs. 3-5) and, if the $A$ and $B$ states are in pure precession, this requires that $\gamma^{\prime}<0$. In other words, the $P_{2}(N)$ (or $R_{2}(N)$ ) line is always to the blue of the $P_{1}(N)$ (or $R_{1}(N)$ ) line. The variation with $N$ of the separation between consecutive rotational lines belonging to the same $F_{i}$ branch is, in this case, very small compared to that of the $\Delta N=0$ combination difference since $|\Delta B| \ll\left|\gamma^{\prime}\right|$. With knowledge that $\gamma^{\prime}$ is negative and larger in magnitude than $\gamma^{\prime \prime}$ and $\Delta B$, one may quickly identify
pairs of $F_{i}$ branches (e.g., $P_{1}$ and $P_{2}$ ) from a set of consecutive excitation spectra. The appropriate $\gamma^{\prime}$ obtained from second differences is substituted into either [1] or [2] to establish an absolute rotational numbering for both branches.

The line positions for each band of the $\Delta v=0$ sequence were fit to standard expressions for the energy levels of a ${ }^{2} \Sigma-{ }^{2} \Sigma$ transition (ref. 16, p. 249) by a nonlinear, weighted, least-squares procedure. The rotational and band origin constants obtained from the fits for the $X$ and $B$ states are listed in Tables $1-4$. In the fits involving the $(0,0)$ and $(1,1)$ bands the $B_{v}{ }^{\prime \prime}$ and $\gamma^{\prime \prime}$ constants were fixed to the values reported in ref. 4 while the $D_{v}{ }^{\prime \prime}$ constant was fixed to the value given in ref. 5. The calculated $B_{v}$ values were obtained by a weighted least-squares fit to the expression
[3] $B_{v}=B_{\mathrm{e}}-\alpha_{\mathrm{e}}(v+1 / 2)+\gamma_{\mathrm{e}}(v+1 / 2)^{2}$
and for $D_{v}$ to the expression
[4] $\quad D_{v}=D_{\mathrm{e}}+\beta_{\mathrm{e}}(v+1 / 2)$
A complete fit of the $\Delta v=-1$ bands was not

Table 3. The $B^{2} \Sigma^{+}$spin-rotation constants as a function of vibrational level in reciprocal centimetres

| $v$ | $\gamma_{0}$ |  |
| :---: | :---: | :---: |
|  | Experimental* | Exp. - cal. $\dagger$ |
| 0 | -0.04581(3) | $-0.00003$ |
| 1 | -0.04607(2) | 0.00002 |
| 2 | -0.04642(6) | -0.00001 |
| 8 | -0.04860(3) | -0.00003 |
| 9 | -0.04897(3) | -0.00000 |
| 12 | -0.05019(3) | 0.00002 |
| 13 | -0.05063(2) | 0.00002 |
| 14 | -0.05115(3) | -0.00006 |
| 15 | -0.05149(6) | 0.00006 |
| *Uncertainties in parentheses represent one standard deviation. <br> $t$ Calculated from the weighted least-squares fit of $\gamma_{6}$ $\gamma_{v}=-0.04562(3)-0.00030(1)(v+1 / 2)-0.0000050(7)$ $(i)+1 / 2)^{2}$. |  |  |

Table 4. The $B^{2} \Sigma^{+}-X^{2} \Sigma^{+}$band origins in reciprocal centimetres

|  | $\mathrm{V}_{0}$ |  |
| :--- | :--- | ---: |
| $v^{\prime}, v^{\prime \prime}$ | Experimental $^{*}$ | Exp. - cal. |
| 0,0 | $18833.139(1)$ | 0.002 |
| 1,1 | $18816.442(1)$ | -0.002 |
| 2,2 | $18799.293(1)$ | -0.002 |
| 8,8 | $18687.260(2)$ | 0.000 |
| 9,9 | $18667.146(1)$ | 0.004 |
| 12,12 | $18604.478(2)$ | 0.003 |
| 13,13 | $18582.841(0)$ | -0.001 |
| 14,14 | $18560.852(1)$ | 0.001 |
| 15,15 | $18538.513(1)$ | 0.002 |
| 0,1 | $18250.296(7)$ | 0.002 |
| 1,2 | $18239.343(3)$ | -0.001 |
| 2,3 | $18227.890(2)$ | 0.001 |
| 3,4 | $18215.937(2)$ | 0.000 |
| *Uncertainties in parentheses represent one standard |  |  | deviation.

performed due to the insufficient number of lines recorded. Instead, the $\Delta v=-1$ band origins were determined by averaging the difference

$$
v_{o}\left(v^{\prime}, v^{\prime \prime}\right)=v(N)_{\text {measured }}-F_{i}^{\prime}\left(N^{\prime}\right)+F_{j}^{\prime \prime}\left(N^{\prime \prime}\right)
$$

using the rotational constants obtained from the $\Delta v=0$ band fits.
The $\gamma^{\prime}$ constants for the $B$ state are listed in Table 3. The calculated $\gamma_{v}$ constants were obtained by a weighted least-squares fit to the empirical expression
[5] $\quad \gamma_{v}=\gamma_{1}+\gamma_{2}(v+1 / 2)+\gamma_{3}(v+1 / 2)^{2}$
For the $X$ state we did not observe any significant vibrational dependence of the spin-rotation constant. From a trial fit of the $(13,13)$ band, a $\gamma^{\prime \prime}=$ $0.0012 \pm 0.0004 \mathrm{~cm}^{-1}$ was determined. A high pre-
cision value for $\gamma^{\prime \prime}=0.001287 \pm 0.000007 \mathrm{~cm}^{-1}$ was measured for $v^{\prime \prime}=0$ by Bernath et al. (17) using the technique of intermodulation spectroscopy on the $A-X(0,0)$ band. Therefore, we concluded that the vibrational dependence of $\gamma^{\prime \prime}$ can be ignored and its value was fixed to 0.0013 in all subsequent fits.

## 2. Vibrational Analysis

The vibrational constants for the $X$ and $B$ states were obtained from a weighted least-squares fit of the $\Delta v=0$ and -1 band origins to the expression
[6] $\quad T_{v}=T_{\mathrm{e}}{ }^{\prime}+\omega_{\mathrm{e}}{ }^{\prime}\left(v^{\prime}+1 / 2\right)$

$$
\begin{aligned}
& -\omega_{\mathrm{e}}{ }^{\prime \prime}\left(v^{\prime \prime}+1 / 2\right)-\omega_{\mathrm{e}}{ }^{\prime} x_{\mathrm{e}}^{\prime}\left(v^{\prime}+1 / 2\right)^{2} \\
& +\omega_{\mathrm{e}}^{\prime \prime} x_{\mathrm{e}}^{\prime \prime \prime}\left(v^{\prime \prime}+1 / 2\right)^{2}+\omega_{\mathrm{e}}^{\prime} y_{\mathrm{e}}^{\prime}\left(v^{\prime}+1 / 2\right)^{3} \\
& -\omega_{\mathrm{e}}^{\prime \prime} y_{\mathrm{e}}^{\prime \prime \prime}\left(v^{\prime \prime}+1 / 2\right)^{3}
\end{aligned}
$$

A summary of the molecular constants for both states is presented in Table 5.
3. Estimates of $\alpha_{e}, D_{e}, \beta_{e}, \gamma_{e}$, and $\omega_{e} y_{e}$

In addition to the molecular constants listed in Table 5 obtained from the various fits for the $X$ and $B$ states, estimates are also included for the con$\operatorname{stants} \alpha_{e}, D_{\mathrm{e}}, \boldsymbol{\beta}_{\mathrm{e}}, \gamma_{\mathrm{e}}$, and $\omega_{\mathrm{e}} y_{\mathrm{e}}$ for both states. Because these particular constants can be expressed in terms of $B_{e}, \alpha_{e}, \omega_{e}$, and $\omega_{e} x_{e}$, using relationships derived by Dunham and certain assumed model potentials, the estimates, especially for $\alpha_{\mathrm{e}}, \beta_{\mathrm{e}}$, and $\omega_{\mathrm{e}} y_{\mathrm{e}}$, provide a means to determine whether the fitted constants have reasonable values.

The $\alpha_{e}$ and $D_{e}$ constants were estimated using the standard Pekeris (18) and Kratzer (ref. 16, p. 103) relations, respectively. The $\beta_{\mathrm{e}}$ constants were calculated using the Dunham expression (19)
[7] $\beta_{\mathrm{e}} \simeq-Y_{12}=\left(12 B_{\mathrm{e}}{ }^{4} / \omega_{\mathrm{e}}{ }^{3}\right)$

$$
\times\left(19 / 2+9 a_{1}+9 a_{1}^{2} / 2-4 a_{2}\right)
$$

The Dunham $a_{1}$ and $a_{2}$ coefficients were computed using [8] and [9] respectively, listed below.
[8] $a_{1}=-\alpha_{e} \omega_{e} / 6 B_{e}{ }^{2}-1$
[9] $a_{2}=5 a_{1}{ }^{2} / 4-2 \omega_{\mathrm{e}} x_{\mathrm{e}} / 3 B_{\mathrm{e}}$
But in order to calculate $\gamma_{e}$ and $\omega_{e} y_{e}$ from [10] and [11], respectively,
[10] $\gamma_{\mathrm{e}} \simeq Y_{21}=\left(6 B_{\mathrm{e}}{ }^{3} / \omega_{e}{ }^{2}\right)\left[5+10 \mathrm{a}_{1}\right.$

$$
\left.-3 a_{2}+5 a_{3}-13 a_{1} a_{2}+15\left(a_{1}^{2}+a_{1}^{3}\right) / 2\right]
$$

$$
\begin{align*}
\omega_{\mathrm{e}} y_{\mathrm{e}}= & Y_{30}=\left(B_{\mathrm{e}}{ }^{2} / 2 \omega_{\mathrm{e}}\right)\left[10 a_{4}-35 a_{1} a_{3}\right.  \tag{11}\\
& \left.-17 a_{2}{ }^{2} / 2+225 a_{1}{ }^{2} a_{2} / 4-705 a_{1}{ }^{4} / 32\right]
\end{align*}
$$

it was necessary to obtain expressions for the

Table 5. Equilibrium molecular constants for the $B^{2} \Sigma^{+}$and $X^{2} \Sigma^{+}$states in reciprocal centimetres*

|  | $X^{2} \Sigma^{+}$ |  | $B^{2} \Sigma^{+}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Fit | Calculated | Fit | Calculated |
| $T_{\text {e }}$ | 0 | - | 18841.309(3) | - |
| $\omega_{c}$ | 588.633(36) | - | 572.405(36) | - |
| $\omega_{c} x_{c}$ | $2.908(13)$ | - | $3.143(13)$ | - |
| $\omega_{c} y_{\text {e }}$ | 0.0081 (15) | 0.010(2) | $0.0095(15)$ | 0.012(2) |
| $B_{\text {c }}$ | $0.343715(7)$ | -0.02300 | $0.342604(7)$ | . - |
| $\alpha_{c}$ | $0.002453(5)$ | 0.002300(6) | $0.002630(6)$ | 0.002494(6) |
| $\gamma_{c}$ | $0.0000062(4)$ | $0.0000069(14)$ | $0.0000063(4)$ | $0.0000073(16)$ |
| $D_{\text {c }} \times 10^{7}$ | 4.21 (5) | $4.6878(6)$ | 4.66(3) | 4.9094(7) |
| $\beta_{c} \times 10^{9}$ | 6.13(42) | 0.11 (14) | 3.93 (56) | 0.69 (17) |
| $R_{e}(\AA)$ | - | 1.952 | - | 1.955 |

*Uncertainties in parentheses represent one standard deviation.

Dunham $a_{3}$ and $a_{4}$ coefficients. Approximate expressions for $a_{3}$ and $a_{4}$ in terms of $a_{1}$ and $a_{2}$ were derived by Jordan et al. (20) from a method, using Padé approximants, to construct potential curves for ionic molecules. The expressions for $a_{3}$ and $a_{4}$, obtained from ref. 20, are

$$
\begin{array}{ll}
{[12]} & a_{3} \simeq 2 a_{1} a_{2}-a_{1}{ }^{3} \\
{[13]} & a_{4} \simeq a_{2}{ }^{2}+a_{1}{ }^{2} a_{2}-a_{1}{ }^{4}
\end{array}
$$

With the exception of the $\beta_{\mathrm{e}}$ constants, reasonable agreement is obtained between estimated and fitted constants.

## Discussion

The rotational analysis of the $B-X$ system indicates that the $A^{2} \Pi$ and $B^{2} \Sigma^{+}$states are in pure precession. From the value of $p=-0.04454 \mathrm{~cm}^{-1}$ reported in ref. 5 for the $v=0$ level of the $A$ state, the ratio of $\gamma_{0} / p_{0}\left(\gamma=-0.04581 \mathrm{~cm}^{-1}\right.$ for $v=0$ of the $B$ state) is 1.029 and the pure precession relation holds to within $3 \%$. A value of $l=1.04$ for the $B$ state is obtained by substituting $\gamma_{1}$ from [5], $T_{\mathrm{e}}$ for the $B$ state, and the $A$ state $B_{\mathrm{e}}, T_{\mathrm{e}}$, and spin-orbit constants from ref. 4 into the vibration-independent term of the Van Vleck pure precession relation. Thus, it seems that the $A^{2} \Pi$ and $B^{2} \Sigma^{+}$states originate from the $3 p$ complex of the calcium ion as suggested in refs. 5 and 21.
The $X$ state vibrational constants reported in ref. 5 were poorly determined since they incorporated bandhead data from Johnson (1) in their weighted least-squares fit. It is not surprising that there are differences between the two sets of constants, illustrating that raw bandhead data from ref. 1 must be treated with caution.

Franck-Condon factors were calculated for all sequences up to $v=20$ using standard RKR (22) and FC (23) computer programs. The FranckCondon factors for the $\Delta v=0$ and $\pm 1$ sequences
are displayed in Table 6. As expected, the $\Delta v=0$ sequence is the most intense followed by weaker $\Delta v= \pm 1$ sequences. The ratio

$$
\frac{I_{\Delta v=+1}+I_{\Delta v=-1}}{2 I_{\Delta v=0}}
$$

changes from 0.0023 for $v=1$ to 0.30 at $v=19$ so the vibrational structure becomes appreciably less diagonal as $v$ increases.

We observed no sharp bandhead structure for the $\Delta v=-1$ sequence, in contrast to the $\Delta v=0$ sequence. A closer look at the rotational constants for both states explains this observation. The bandhead to band origin separation is inversely proportional to $\Delta B$. For the $\Delta v \geq 0$ sequences $\Delta B$ is always negative, resulting in red degraded rota-

Table 6. Franck-Condon factors for the $\Delta v=-1,0$, and +1 sequences of the $B^{2} \Sigma^{+}-X^{2} \Sigma^{+}$transition

| $v^{\prime \prime}$ | $q_{v^{\prime \prime}-1, v^{\prime \prime} \times 10^{2}}$ | $q_{v^{\prime \prime}, v^{\prime \prime}}$ | $q_{v^{\prime \prime}+1, v^{\prime \prime} \times 10^{2}}$ |
| :---: | :---: | :---: | :---: |
| 0 | - | 0.999 | 0.137 |
| 1 | 0.134 | 0.995 | 0.333 |
| 2 | 0.321 | 0.991 | 0.596 |
| 3 | 0.566 | 0.985 | 0.938 |
| 4 | 0.877 | 0.977 | 1.37 |
| 5 | 1.26 | 0.968 | 1.89 |
| 6 | 1.72 | 0.956 | 2.52 |
| 7 | 2.25 | 0.943 | 3.26 |
| 8 | 2.88 | 0.928 | 4.12 |
| 9 | 3.59 | 0.910 | 5.10 |
| 10 | 4.40 | 0.890 | 6.22 |
| 11 | 5.29 | 0.867 | 7.46 |
| 12 | 6.27 | 0.842 | 8.83 |
| 13 | 7.35 | 0.815 | 10.3 |
| 14 | 8.51 | 0.785 | 11.9 |
| 15 | 9.74 | 0.753 | 13.7 |
| 16 | 11.1 | 0.718 | 15.5 |
| 17 | 12.4 | 0.681 | 17.4 |
| 18 | 13.8 | 0.642 | 19.4 |
| 19 | 15.3 | 0.601 | 21.4 |
| 20 | 16.8 | 0.558 | - |

tional branches. This is not the case for the $\Delta v<0$ sequences. For a given sequence one can write

$$
\Delta B=\Delta B_{\mathrm{e}}-\alpha_{\mathrm{e}}^{\prime} \Delta v-\Delta \alpha_{\mathrm{e}}\left(v^{\prime \prime}+1 / 2\right)+\ldots
$$

For the $B-X$ system $\Delta \alpha_{\mathrm{e}}>0$ and $\Delta B_{\mathrm{e}}<0$. The above equation reveals that for $\Delta v<0$ sequences, a sign reversal of $\Delta B$ will occur at some value of $v$. Because $\alpha_{\mathrm{e}}{ }^{\prime}$ is slightly larger than $\Delta B_{\mathrm{e}}$, the reversal will occur at fairly low $v$. Thus, the bandhead pattern in a $\Delta v<0$ sequence is violet shaded heads for low $v$, becoming headless at a particular $v$, followed by red shaded heads for high $v$. For the $\Delta v=-1$ sequence, the bandhead to band origin separation for the ( 0,1 ) band is already fairly large, approximately $15 \AA$, and increases steadily to infinity as $\Delta B$ approaches zero. This occurs near the $(7,8)$ band. From then on, the bandheads are shaded to the red and a steady decrease in bandhead to band origin separation is expected. The bandhead intensity is diminished considerably because the population of the very high $N$ levels where the head occurs is essentially zero. This, combined with closely spaced sequence bands, results in the nonoccurrence of a sharp bandhead structure.

## Acknowledgements

This research was supported by grants from the National Science Foundation (CHE-78-10178) and the Air Force Office of Scientific Research (AFOSR-76-3056). P.F.B. was supported, in part, by a Natural Sciences and Engineering Research Council of Canada postgraduate scholarship. We thank J. Manni and J. Joens, for their contributions to the early stages of this project, and P. Pappas, for his helpful suggestions on operation of the rhodamine 110 dye laser near $5300 \AA$.

1. R. C. Johnson. Proc. R. Soc. London, Ser. A. 122, 161 (1929)
2. A. Harvey. Proc. R. Soc. London, Ser. A. 133. 336 (1931).
3. R. W. Field. D. O. Harris, and T. Tanaka. J. Mol. Spectrosc. 57, 107 (1975).
4. J. Nakagawa, P. J. Domaille, T. C. Steimle, and D. O. Harris. J. Mol. Spectrosc. 70, 374 (1978).
5. P. F. Bernath and R. W. Field. J. Mol. Spectrosc. In press.
6. B. S. Mohanty and K. N. Upadhya. Indian J. Pure Appl. Phys. 5, 523 (1967)
7. L. K. Khanna and V. S. Dubey. Indian. J. Pure Appl. Phys. 11, 444 (1973).
8. D. P. Nanda and B. S. Mohanty. Curr. Sci. 39, 300 (1970).
9. W. Demtröder. In Case studies in atomic physics. Vol. 6. Edited by M. R. C. McDowell and E. W. McDaniels. North-Holland, Amsterdam. The Netherlands. 1976.
10. C. Linton. J. Mol. Spectrosc. 69, 351 (1978).
11. J. B. West. R. S. Bradford. J. D. Eversole, and C. R. Jones. Rev. Sci. Instrum. 46. 164 (1975).
12. D. L. Hildenbrand. Adv. High Temp. Chem. 1, 193 (1966).
13. T. Klang. R. C. Estler, and R. N. Zare. J. Chem. Phys. 70, 5925 (1979).
14. S. Gerstenkorn and P. Luc. Atlas du spectre d'absorption de la molécule d'iode. CNRS, Paris, France. 1978.
15. S. Gerstenkorn and P. Luc. Rev. Phys. Appl. 14, 791 (1979).
16. G. Herzberg. Spectra of diatomic molecules. 2nd ed. Van Nostrand Reinhold Co.. New York. NY. 1950.
17. P. F. Bernath, P. G. Cummins. I. Renhorn, and R. W. Field. Chem. Phys. Lett. In press.
18. C. L. Pekeris. Phys. Rev. 45, 98 (1934).
19. J. L. Dunham. Phys. Rev. 41, 721 (1932).
20. K. D. Jordan. J. L. Kinsey, and R. Silbey. J. Chem. Phys. 61. 911 (1974).
21. P. J. Dagdigian. H. W. Cruse, and R. N. Zare. J. Chem. Phys. 60. 2330 (1974).
22. R. J. LeRoy. Private communication.
23. R. N. Zare. J. Chem. Phys. 40. 1934 (1964).

## Appendix

In addition to the blended lines within a band, a few blends between bands were present. The $P_{2}(6)$ line of the $(1,1)$ band was blended with $P_{1}(28)$ of the $(0,0)$ band. For the $(1,2)$ and $(2,3)$ bands the only inter-band blends were those of $R_{2}(1)$ to $R_{2}(4)$ lines of the (1,2) band with $R_{2}(17)$ to $R_{2}(20)$ of the ( 2,3 ) band.

The units are reciprocal centimetres and it is recommended (15) that a correction of $0.0056 \mathrm{~cm}^{-1}$ be subtracted from all lines. The numbers in parentheses are (observed - calculated) in units of $10^{-3}$ reciprocal centimetres. The asterisk indicates a blended line.

Table A1. Measured line positions

| Band | $N$ | $R_{1}$ | $R_{2}$ | $P_{1}$ | $P_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(0,0)$ | 10 | $18840.256(2)$ | $18840.792(-2)$ | - | - |
|  | 11 | $18840.886(1)$ | $18841.473(0)^{*}$ | $18825.232(-6)$ | -- |
|  | 12 | $18841.507(-7)^{*}$ | $18842.147(-2)^{*}$ | $18824.501(-2)$ | $18825.041(-5)$ |
|  | 13 | $18842.147(6)^{*}$ | $18842.818(-5) *$ | $18823.767(0)$ | $18824.355(-2)$ |
|  | 14 | 18842.764(-1)* | $18843.496(2)$ | $18823.026(-2)$ | $18823.665(0)$ |
|  | 15 | 18843.390(4) | - | - | 18822.972(1)* |
|  | 16 | - | - | - | $18822.281(6) *$ |
|  | 17 | - | - | $18820.799(2)$ | - |
|  | 18 | - | 18851.339(0) | 18813.984(1) | $18820.876(1)$ |
|  | 26 | - | $18851.339(0)$ | $18813.984(1)$ | - |
|  | 27 | - | $18851.972(-3)$ | $18813.216(1)$ | --- |
|  | 28 | $18851.216(-3)$ | 18852.609(1) | 18812.450(5)* | $18813.741(0)$ |
|  | 29 | $18851.799(-3)$ | -- | 18811.669(-3) | $18813.016(0)$ |
|  | 30 | $18852.380(-2)$ | -- | $18810.899(2)$ | $18812.288(0)$ |
|  | 31 | $18852.960(1)$ | - | -- | $18811.558(0)$ |
|  | 34 | - | - | 18807.780(6) | - |
|  | 35 | - | - | $18806.988(0)$ | - |
|  | 36 | -- | - | $18806.202(2)$ | $18807.875(2)$ |
|  | 37 | - | - | $18805.408(-1)$ | $18807.128(-2)$ |
|  | 38 | - | - | - | $18806.386(2)$ |
|  | 39 | - | - | - | $18805.635(-1)$ |
| $(1,1)$ | 5 | 18820. - $^{\text {(2) }}$ | $18820.630(0)$ | $18812.922(3)$ | $18813.133(-1)$ |
|  | 6 | $18820.962(2)$ | $18821.311(-3)$ | $18812.199(-3)$ | 18812.454(-10)* |
|  | 7 | $18821.587(-7)^{*}$ | $18821.995(-1)$ | $18811.481(-1)$ | $18811.787(-4)$ |
|  | 8 | - | $18822.675(1)$ | $18810.760(1)$ | $18811.115(0)$ |
|  | 9 | $18822.855(1)$ | - | $18810.037(3)$ | 18810.438(1) |
|  | 10 | - | - | 18809.314(9)* | $18809.759(2)$ |
|  | 11 | - | - | $18808.574(-1)$ | $18809.075(2)$ |
|  | 12 | 18825.341(3) | 18826.021 | 18807.842(1) | $18808.386(-1)$ |
|  | 13 | 18825.341 (3) | $18826.021(-3)^{*}$ | - | $18807.699(0)$ |
|  | 14 | $18825.951(-1)$ | $18826.684(-1)$ | -- | - |
|  | 15 | $18826.562(-1)$ | $18827.340(-3)$ | - | - |
|  | 16 | $18827.170(-1)$ | 18827.997(-1) | -- | - |
|  | 17 | $18827.773(-2)$ | $18828.648(-2)$ | $18804.135(1)$ | -- |
|  | 18 | $18828.378(1)$ | $18829.301(2)$ | $18803.386(1)$ | - |
|  | 19 | $18828.973(-3)$ | $18829.947(2)$ | $18802.635(2)$ | $18803.514(3)$ |
|  | 20 | $18829.572(1)$ | $18830.587(-1)$ | $18801.881(2)$ | $18802.804(0)$ |
|  | 21 | $18830.165(1)$ | $18831.226(-2)$ | $18801.121(0)$ | $18802.095(1)$ |
|  | 22 | $18830.750(-3)$ | -- | $18800.363(2)$ | 18801.382(1) |
|  | 23 | 18831.336(-3) | - | $18799.597(-2)$ | $18800.668(2)$ |
|  | 24 | - | - | - | 18799.949 (1) |
|  | 25 | - | - | - | 18799.230 (2) |
|  | 28 | - | $18835.617(0)$ | - | - |
|  | 29 | - | $18836.232(1)$ | - | - |
|  | 30 | $18835.351(0)$ | $18836.842(0)$ | - | - |
|  | 31 | $18835.911(0)$ | $18837.448(-1)$ | - | - |
|  | 32 | $18836.468(0)$ | - | -- | - |
|  | 33 | $18837.022(1)$ | - | - | - |
|  | 34 | $18837.570(0)$ | - | - | - |
| $(2,2)$ | 5 | $18803.138(1)$ | 18803.447 (1) | - | - |
|  | 6 | $18803.766(0)$ | $18804.125(2)$ | - | - |
|  | 7 | 18804.392(-1) | $18804.794(-3)$ | - | - |
|  | 8 | $18805.013(-3)$ | 18805.462(-6) | -- | - |
|  | 9 | 18805.629(-7) | - | $18792.916(2)$ | 18793.323 (2) |
|  | 10 | - | - | $18792.190(2)$ | $18792.643(0)$ |
|  | 11 | - - | - | $18791.462(4)$ | $18791.961(0)$ |
|  | 12 | $18807.474(-2)$ | 18808.772(-1) | $18790.726(0)$ | $18791.274(-2)$ |
|  | 13 | $18808.083(0) *$ | $18808.772(-1)$ | $18789.992(2)$ | $18790.587(-1)$ |
|  | 14 | $18808.686(0)$ | $18809.426(2)$ | - | $18789.898(1)$ |
|  | 15 | $18809.287(1)$ | $18810.073(1)$ | - | $18789.203(-1) *$ |
|  | 16 | $18809.886(3)$ |  | - |  |
|  | 21 | - | - | $18783.995(-3)$ | - |

Table Al (Contimued)

| Band | $N$ | $R_{1}$ | $R_{2}$ | $P_{1}$ | $P_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(2,2)$ | 22 | - | - | $18783.236(0)$ | 18784.260(-3) |
|  | 23 | - | - | - | $18783.543(-2)$ |
|  | 24 | - | - | - | $18782.830(5)$ |
| $(8,8)$ | 16 | - | - | $18675.924(4) *$ | 18675.988(-3)* |
|  | 17 | - | - | $18675.169(3)$ | $18675.988(-3)^{*}$ |
|  | 18 | - | - | 18674.404(-3) | $18675.280(-1)$ |
|  | 19 | - | - | $18673.645(2)$ | $18674.566(-1)$ |
|  | 20 | - | $18700.157(-1)$ | $18672.874(1)$ | $18673.846(-2)$ |
|  | 21 | - | $18700.711(0)$ | --- | $18673.122(-1)$ |
|  | 22 | $18700.089(2)$ | $18701.256(-3)$ | - | $18672.392(-2)$ |
|  | 23 | $18700.580(1)$ | $18701.798(-2)$ | - | 18672.3921 |
|  | 24 | $18701.067(2)$ | $18702.334(-2)$ | - | - |
|  | 25 | $18701.546(1)$ | $18702.868(1)$ |  |  |
|  | 26 | $18702.020(0)$ | . | - |  |
|  | 27 | 18702.490 (1) | - | - | - |
|  | 28 | $18702.955(3)$ | - | - | - |
|  | 44 | $18709.540(-3)$ | - | - | - |
|  | 45 | $18709.899(-3)$ | - | - | - |
|  | 46 | $18710.256(1)$ | - | $18651.061(3)$ | - |
|  | 47 | $18710.600(-1)$ | - | 18650.149 (1) | - |
|  | 48 | 18710.941 (1) | - | $18649.231(-1)$ | - |
|  | 49 | $18711.273(0)$ | -- | $18648.307(-4)$ | $18650.737(4)$ |
|  | 50 | $18711.601(1)$ | - | $18647.385(0)$ | $18649.860(4)$ |
|  | 51 | 18711.921 (2) | - | $18646.449(-4)$ | $18648.974(0)$ |
|  | 52 | - | - | - | $18648.086(-1)$ |
|  | 53 | - | - | -- | $18647.195(1)$ |
|  | 54 | - | - | - | $18646.295(-1)$ |
| $(9,9)$ | 11 | 18674.651 | $18674.733(-2)$ | $18659.527(0)$ | 1865-377(-2) |
|  | 12 | $18674.651(0)$ | $18675.328(-1)$ | $18658.799(0)$ | $18659.377(-2)$ |
|  | 13 | $18675.190(1)$ | $18675.912(-4)$ | $18658.069(3) *$ | $18658.694(-2)$ |
|  | 14 | $18675.719(-1)$ | -- | - | $18658.010(3) *$ |
|  | 15 | $18676.243(-2)$ | - | - | 18657.320(7)* |
|  | 31 | - | $18685.481(5)$ | - | - |
|  | 32 | - | $18685.948(-1)$ | - | - |
|  | 33 | - | $18686.414(-2)$ | - | - |
|  | 34 | - | $18686.876(-1)$ | 18640.546(2) | - |
|  | 35 | - | $18687.331(0)$ | 18640.546 (2) | - |
|  | 36 | $18685.901(5)^{*}$ | $18687.781(2)$ | $18639.683(-1)$ | 18640.657(2) |
|  | 37 | $18686.284(-3)$ | 18688.221 (0) | $18638.818(-1)$ | $18640.657(2)$ |
|  | 38 | $18686.671(-1)$ | -- | - | $18639.833(-2)$ |
|  | 39 | $18687.052(2)$ | - | - | $18639.008(-1)$ |
|  | 40 | ( | - | - | $18638.177(-1)$ |
| $(12,12)$ | 10 | $18610.662(3)$ | $18611.247(-3)$ | -- | - |
|  | 11 | $18611.179(-1)$ | $18611.820(-2)$ | - | - |
|  | 12 | $18611.693(-1)$ | $18612.386(-2)$ | - | - |
|  | 13 | $18612.201(-1)$ | $18612.946(-1)$ | - | - |
|  | 14 | $18612.703(1)$ | $18613.500(1)$ | - | - |
|  | 15 | $18613.197(1)$ | $18614.044(0)$ | - | - |
|  | 16 | $18613.687(5)$ | - | - | - |
|  | 35 | - | - | $18577.716(1)$ | - |
|  | 36 | - | - | $18576.833(1)$ | - |
|  | 37 | - | - | $18575.941(-1)$ | $18577.825(2)$ |
|  | 38 | - | - | $18575.045(-1)$ | $18576.982(4)$ |
|  | 39 | -- | - | $18574.144(1)$ | $18576.126(-1)$ |
|  | 40 | - | - | $18573.233(-1)$ | $18575.271(2)$ |
|  | 41 | - | - | - | $18574.402(-3)$ |
|  | 42 | - | - | - | $18573.535(1)$ |
| $(13,13)$ | 2 | $18584.591(-1)$ | 18584.773 (0) | - | - |
|  | 3 | - | $18585.391(-3)$ | - | - |
|  | 4 | $18585.726(2)$ | $18586.010(1)$ | - | - |
|  | 5 | $18586.283(3)$ | $18586.617(1)$ | - | - |

Table A1 (Continued)

| Band | $N$ | $R_{1}$ | $R_{2}$ | $P_{1}$ | $P_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(13,13)$ | 6 | $18586.827(-1)$ | $18587.215(-1)$ | - | - |
|  | 7 | 18587.370 (1) | $18587.808(-1)$ | $18578.175(2)$ | $18578.513(1)$ |
|  | 8 | $18587.904(1)$ |  | $18577.475(0)$ | $18577.865(-1)$ |
|  | 9 | - | - | $18576.769(-1)$ | $18577.209(-4)$ |
|  | 10 | 18589.465 | 18589.547 (1) | $18576.059(1)$ | $18576.550(-3)$ |
|  | 11 | $18589.465(2)$ | $18590.110(-1)$ | $18575.341(2)$ | $18575.885(-1)$ |
|  | 12 | $18589.969(0)$ | $18590.667(-1)$ | $18574.612(-1)$ | $18575.213(1)$ |
|  | 13 | $18590.467(0)$ | 18591.220 (1) | $18573.875(-6)^{*}$ | $18574.529(-2)$ |
|  | 14 | $18590.958(0)$ | - | $18573.148(7)^{*}$ | $18573.853(10)^{*}$ |
|  | 15 | $18591.443(1)$ | $18592.299(1)$ | $18572.398(3)$ | $18573.148(-1)^{*}$ |
|  | 16 | 18592. ${ }^{\text {- }}$ | - | - | $18572.448(0)$ |
|  | 17 | $18592.388(0)$ | 18593.855 | - | - |
|  | 18 | $18592.848(-2)$ | $18593.855(-6)$ | - | - |
|  | 19 | $18593.304(-1)^{*}$ | $18594.367(-1)$ | - | - |
|  | 20 | $18593.751(-1)$ | $18594.867(0)$ | - | - |
|  | 21 | $18594.191(-1)$ | 18595.359 (0) | - | - |
|  | 22 | $18594.624(-1)$ | - | - | - |
|  | 23 | $18595.050(0)$ | - | - | - |
|  | 24 | $18595.469(1)$ | -- | - | - |
|  | 28 | $18597.065(1)$ | $18598.590(-5)^{*}$ | - | - |
|  | 29 | $18597.445(1)$ | $18599.028(1)$ | - | - |
|  | 30 | $18597.816(-1)$ | $18599.453(2)$ | - | - |
|  | 31 | $18598.181(-1)$ | - | - | - |
|  | 32 | 18598.541 (2) | - | -- | - |
|  | 33 | $18598.888(-1)$ | - | - | - |
|  | 34 | $18599.229(-2)$ | - | - | - |
|  | 35 | $18599.564(-1)$ | -- | - | - |
|  | 40 | -- | - | $18551.507(0)$ | - |
|  | 41 | - | - | $18550.582(0)$ | - |
|  | 42 | - | - | $18549.649(-1)$ | - |
|  | 43 | - | 18604.581(0) | - | 18550.921 (1) |
|  | 44 | - | $18604.581(0)$ | - | ( |
|  | 45 | - | $18604.888(0)$ | - | - |
|  | 46 | - | 18605.190 (3) | - | - |
|  | 47 | - | 18605.479 (1) | - | - |
|  | 48 | - | $18605.760(-1)$ | - | - |
|  | 49 | -- | $18606.035(0)$ | - | - |
|  | 50 | - | $18606.300(-1)$ | - | - |
| $(14,14)$ | 8 | - | - |  | 1855. - |
|  | 9 | - | - | 18554.811 (1) | $18555.257(0)$ |
|  | 10 | - | - | 18554.101 (2) | $18554.602(3)$ |
|  | 11 | - | - | $18553.381(-1)$ | $18553.934(0)$ |
|  | 12 | - | - | $18522.657(0)$ | $18533.258(-3)$ |
|  | 13 | - | - | - | $18552.579(-3)$ |
|  | 14 | - | - | $18551.192(7)^{*}$ | 18551.192(-9)* |
|  | 15 | - | - | $18550.444(5)^{*}$ | $18551.192(-9)^{*}$ |
|  | 21 | - | $18573.188(-1)^{*}$ | - | 18551.192(-9)* |
|  | 22 | 18572-840(-2) | $18573.651(-10) *$ | - | - |
|  | 23 | $18572.840(-2)$ | $18574.126(0)$ | - | - |
|  | 24 | $18573.246(-1)^{*}$ | $18574.582(-1)$ | - | - |
|  | 25 | $18573.651(7)^{*}$ | $18575.032(0)$ | - | - |
|  | 26 | $18574.034(2)$ | $18575.480(7)^{*}$ | - | - |
|  | 27 | $18574.414(1)$ | $18575.905(-2)^{*}$ | - | - |
|  | 28 | $18574.788(1)$ | $18576.328(-5)$ | -- | - |
|  | 29 | $18575.152(0)$ | 18577-164(4)* | - | - |
|  | 30 | $18575.500(-9)^{*}$ | 18577.164(4)* | - | - |
|  | 31 | $18575.866(7)^{*}$ | $18577.564(2)$ | - | - |
|  | 32 | $18576.196(-4)$ | $18577.956(0)$ | - | - |
|  | 33 | $18576.533(-1)$ | 18578.344 (2) | - | - |
|  | 34 | - |  | - | - |
|  | 35 | $18577.164(-12) *$ | - | - | - |

Table A1 (Concluded)

| Band | $N$ | $R_{1}$ | $R_{2}$ | $P_{1}$ | $P_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $(15,15)$ | 6 | - | 18542.810 (1) | - | - |
|  | 7 | 18542.943 (2) | $18543.389(1)$ | - | - |
|  | 8 | $18543.459(0)$ | - | - | - |
|  | 9 | $18543.967(-3)^{*}$ | $18544.521(-2)^{*}$ | - | - |
|  | 10 | $18544.476(3) *$ | $18545.078(-1)$ | - | - |
|  | 11 | $18544.968(-1)$ | $18545.625(-2)$ | - | - |
|  | 12 | $18545.453(-3)$ | $18546.167(-1)$ | - | - |
|  | 13 | $18545.937(1)$ | $18546.703(3)$ | - | - |
|  | 14 | 18546.410 (2) | - | - | - |
|  | 16 | (8) | - | 18527.380 (1) | - |
|  | 17 | - | - | 18526.618 (0) | $18527.491(1)$ |
|  | 18 | - | - | $18525.847(-2)$ | $18526.776(2)$ |
|  | 19 | - | - | $18525.072(0)$ | $18526.049(-1)$ |
|  | 20 | - | - | - | 18525.319 (0) |
|  | 21 | - | - | - | $18524.580(-1)$ |
| $(0,1)$ | 8 | - | $18256.750(-12)$ | - | - |
|  | 9 | 18256.990 (-6) | $18257.485(-4)$ | - | - |
|  | 10 | $18257.678(-1)$ | 18258.227 (7) | - | - |
|  | 11 | $18258.381(17)$ |  | - | - |
| $(1,2)$ | 1 | $18240.658(5) *$ | $18240.773(3) *$ | - | - |
|  | 2 | $18241.315(4)$ | $18241.471(-4)^{*}$ | - | - |
|  | 3 | $18241.971(0)$ | $18242.176(-7)^{*}$ | - | - |
|  | 4 | $18242.631(-2)$ | $18242.875(-17)^{*}$ | - | - |
|  | 5 | $18243.298(1)$ | $18243.601(-3)$ | - | - |
| $(2,3)$ | 5 | $18231.806(0)$ | $18232.118(3) *$ | - | - |
|  | 6 | $18232.466(-2)$ | $18232.823(2) *$ | - | $18224.038(0)$ |
|  | 7 | $18233.124(-1)$ | $18233.531(2)$ | $18223.089(-1)$ | $18223.404(2)$ |
|  | 8 | $18233.784(-2)^{*}$ | $18234.235(-3)$ | - | - |
|  | 9 | - | $18234.948(-1)$ | - | - |
|  | 17 | 18240.4890) | $18240.697(3)$ | - | - |
|  | 18 | $18240.489(0)$ | $18241.415(-4)$ | - | - |
|  | 19 | $18241.167(-1)$ | $18242.147(2)^{*}$ | - | - |
|  | 20 | $18241.847(-1)$ | $18242.876(4) *$ | - | - |
|  | 21 | $18242.526(-3)$ | - | -- | - |
|  | 22 | $18243.210(-1)$ | - | - | - |
| $(3,4)$ | 10 | $18223.089(8) *$ | $18223.636(3)$ | - | - |
|  | 11 | 18223.744 (6) | $18224.338(0)$ | - | - |
|  | 12 | $18224.398(2)$ | - | - | - |
|  | 21 | - | 18231.449 (0) | - | - |
|  | 22 | 7 | $18232.165(-1)$ | - | - |
|  | 23 | $18231.708(3)$ | $18232.882(-2)$ | - | - |
|  | 24 | $18232.378(2)$ | $18233.601(-2)$ | - | - |
|  | 25 | 18233.048 (1) | 18234.321(-1) | - | - |
|  | 26 | 18 233.724(5) | - | - | - |
|  | 27 | 18234.395(3) | - | - | - |

*Blended line.

