Refractive index of air: new equations for the visible and near infrared

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The precision of modern length interferometry and geodetic surveying far exceeds the accuracy, which is ultimately limited by the inadequacy of currently used equations for the refractive index of the atmosphere. I have critically reviewed recent research at the National Physical Laboratory, the International Bureau of Weights and Measures, and elsewhere that has led to revised formulas and data for the dispersion and density of the major components of the atmosphere. I have combined selected formulas from these sources to yield a set of equations that match recently reported measurements to within the experimental error, and that are expected to be reliable over very wide ranges of atmospheric parameters and wavelength.

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1. Introduction

An accurate knowledge of the refractive index of air is essential to precise length interferometry or geodetic surveying. Where overall uncertainties of approximately 1 part in 10^7 are sought, the refractive index should be known to a few parts in 10^8. The equations currently commonly used in physics to derive the refractive index from the physical parameters of the atmosphere (primarily the temperature, pressure, humidity, and CO_2 content) are those of Edlen.[1] However, in geodetic surveying an older set of equations given by Edlen[2] is still formally recommended, although the later equations[1] are often used. Both these sets of equations were expressed in superseded units; they also embody very old measurements, some of which are known to be in error, and they assume obsolete values of several physical constants and the use of superseded temperature scales.

Since Edlen's research, new measurements of dispersion and of absolute refractive index have been made, and some data have been reassessed. Recent absolute measurements[3] of the refractive index have demonstrated errors of several parts in 10^7 in Edlen's

research. Some of these new results have been incorporated in a recent revision by Birch and Downs[3] (corrected in Ref. 4), with greatly improved fits to the data at a wavelength of 633 nm. However, this study was restricted to conditions likely to occur in a controlled laboratory. The International Association of Geodesy (IAG) has established a Working Group to develop equations better suited to the very wide ranges of atmospheric conditions (and wavelengths) required in geodetic surveying. As part of that effort, I have developed a new set of equations based on more recent equations for density and for dispersion in the visible and the near-infrared regions. These equations agree with the measurements of Birch and Downs[4] to 1 part in 10^8, which is within their experimental uncertainty. Hou and Thalmann[5] have reported experimental results in good agreement with Ref. 4. However, precise modern interferometric refractometers have been found[6] to have uncertainties (at the 1σ level) of $1–3 \times 10^{-8}$ and disagree by approximately the same amount when the same sample of air is measured. These results suggest that absolute uncertainties of better than 5×10^{-8} are unlikely to be achievable by direct measurement, especially over long or nonuniform paths. Nevertheless, it is worthwhile to improve the equations for the indirect calculation of refractive index so that such calculations do not contribute additional uncertainty to the results.

Following Owens,[7] my approach has been to determine the refractivity (the departure of the refractive index from unity) of each component of moist air (i.e.,

dry air and water vapor) at some standard conditions, and the density of each component, relative to its density at the standard conditions. I then multiply the standard refractivity by the relative density to give a partial refractivity, and I combine the results for each component. However, in contrast to Owens, I have adopted the recently developed International Bureau of Weights and Measures (BIPM) equation for the density of moist air. I also use an equation from Peck and Reeder[8] for the refractivity of dry air, and an extension of Erickson's equation for the relative refractivity of water vapor,[9] together with a correction to the absolute value of the refractivity of pure water vapor given by Barrell and Sears.[10] I have also compared the results of simply adding the component refractivities with the results of using the Lorentz–Lorenz relation.

2. Dispersion of Standard Air and Standard Water Vapor

Edlen adopted the refractivity equations that were derived by Barrell and Sears[10] in 1939 from their measurements on dry air and moist air. Birch and Downs[11] found discrepancies between their measurements and calculated refractivities that suggested that the humidity had probably been underestimated by Barrell and Sears, so that the refractivity of water vapor at a given density had been underestimated; they proposed a correction for this error. Measurements by Beers and Doiron[12] confirmed this correction. Alternative dispersion equations for dry air and water vapor have been proposed by several authors[8,13–15] on the basis of more recent measurements. It is my opinion that the most useful of these, because of their large wavelength range and better fit in the near infrared, are those of Peck and Reeder.[8]

I define standard air as dry air at 15 °C, 101,325 Pa, and with 450 ppm (parts in 10^6) CO_2 content, as suggested by Birch and Downs. However, instead of using Edlen's dispersion equation, I use that developed by Peck and Reeder[8] from their data over the wavelength range 230–1690 nm, because it fits the measurements in the infrared better than Edlen's equation. Hill[16] has pointed out that this substitution requires a substantial departure from Erickson's data at 644 nm. The differences between the dispersion equations, although large compared with the internal uncertainties of their fits to the data, are small in absolute terms, in fact less than 0.5×10^{-8}. Because experimental data on absolute refractivities are uncertain by much more than this (see below), the choice of equation is not easy; I have selected the equation of Peck and Reeder on the grounds that it incorporates later experimental data than was available to Edlen. Additional support for this selection comes from precise measurements by Velsko and Eimerl,[17] which yield the dispersion directly, rather than as a difference of two measured refractivities. As shown by Peck,[18] the dispersion between 532 and 1064 nm is better fitted by the Peck and Reeder equation[8] than by that of Edlen.[1]

Following Birch and Downs,[3,4] I have amended these equations to allow for the current International Temperature Scale (ITS 90), which differs from the International Practical Temperature Scale (IPTS 48) presumably used by Peck and Reeder by 9 mK at 15 °C. Similarly, I have further adjusted the Peck and Reeder equation for the change in nominal CO_2 content from $xc = 300$ ppm to $x_c = 450$ ppm. The net change in the numerators in Eq. (1) below is therefore +50 ppm relative to the published values. There is no evidence in the paper by Peck and Reeder that they had adjusted their results to the IPTS 68 temperature scale, but if they had, the correction factor would have been +34 ppm; the effect on the refractive index is much less than 1×10^{-9}. I have also included a correction for departures from 450 ppm of CO_2, because there is strong evidence[11] of substantial variations in CO_2 content in enclosed spaces and of secular changes in the general atmospheric level. This correction factor was first derived by Edlen[1] from Cuthbertson's data[19] at visible wavelengths; I have adopted a slightly different value given by Birch and Downs[4] at 633 nm. I have compared the refractivity of CO_2 implied by Edlen's correction, applied to our calculated refractivity of dry air with 450 ppm of CO_2, with the predictions based on dispersion equations for CO_2 given by Old et al.[20] and by Simmons,[21] over the range 360–2500 nm; the maximum differences in refractive index are less than 1×10^{-8} for CO_2 contents up to 600 ppm. The resulting equations for the refractivities of standard air at 15 °C, 101,325 Pa, and 0% humidity, with 450 ppm (n_{as}) or x_c ppm of CO_2 (n_{axs}) are

$$10^8(n_{as} - 1) = k_1/(k_0 - \sigma^2) + k_3/(k_2 - \sigma^2), \qquad (1)$$

$$(n_{axs} - 1) = (n_{as} - 1)[1 + 0.534 \times 10^{-6}(x_c - 450)], \qquad (2)$$

where σ is the wave number (reciprocal of the vacuum wavelength) in inverse micrometers and the constants are defined in Appendix A.

For pure water vapor at 20 °C and 1333 Pa, the standard conditions assumed by Barrell and Sears,[10] I follow Owens[7] in combining the absolute value of the refractive index given in Ref. 10 and the relative dispersion equation of Erickson,[9] but I use the ITS 90 temperature scale and modern (SI) units. However, as described below, I had to scale this equation to fit the final results to the modern measured values, because of the apparent error[11,12] in the absolute refractivity of water vapor at the standard conditions.

For water vapor at the standard conditions defined by Barrell and Sears (20 °C, 1333 Pa), I found

$$10^8(n_{ws} - 1) = cf(w_0 + w_1\sigma^2 + w_2\sigma^4 + w_3\sigma^6), \qquad (3)$$

where cf (=1.022) is the correction factor one finds by fitting the calculations to the measurements, as mentioned above, and again the constants are defined in Appendix A. Erickson's equation was based on measurements over the range 390–644 nm. An

equation for the refractive index of water vapor that is claimed to agree with recent experimental data was reported by Schiebener *et al.*[22]; however, it relates primarily to liquid water and to steam, and its claimed accuracy is at best 5×10^{-6} in refractive index. I have confirmed that the *dispersion* or relative refractivity given by this equation agrees very closely with Erickson's equation, and that the differences in absolute values are less than 2×10^{-7} throughout the wavelength range 350–1200 nm, which is better than the claimed uncertainty. I therefore consider that Erickson's equation, although based on data in the visible and ultraviolet, is valid in the near infrared. Further research on the dispersion of water vapor in the near infrared would be useful.

3. Density of Moist Air and Its Components

Edlen's equations[1] have been modified by Birch and Downs[3,4] to take account of changes in the temperature scale, the use of SI units, and a new equation for the density (known as the BIPM 1981/91 density equation), that was reported by Giacomo[23] and revised by Davis.[24] The BIPM equation below incorporates the latest values of physical constants such as the gas constant, recent measurements of the enhancement factor for water vapor, the compressibility of air, and the effect of varying concentrations of CO_2.

$$\rho = (pM_a/ZRT)[1 - x_w(1 - M_w/M_a)], \quad (4)$$

where $M_a = 10^{-3}[28.9635 + 12.011 \times 10^{-6}(x_c - 400)]$ kg/mol is the molar mass of dry air containing x_c ppm of CO_2, $M_w = 0.018015$ kg/mol is the molar mass of water vapor, T is the temperature in degrees Kelvin, $R = 8.314510$ J mol^{-1} K^{-1} is the gas constant, p is the total pressure in pascals, x_w is the molar fraction of water vapor in moist air, and Z is the compressibility of the moist air, which is defined in Appendix A in terms of the following parameters: svp $= \exp(AT^2 + BT + C + D/T)$ Pa is the saturation vapor pressure of water vapor in air at temperature T, over liquid water (if the vapor is over ice, see Appendix C), h is the fractional humidity (between 0 and 1), $f = \alpha + \beta p + \gamma t^2$ is the enhancement factor of water vapor in air, t is the temperature in $t = T - 273.15$, and $x_w = fh$ svp$/p$. (Note that partial pressure p_w of water vapor is just h svp.) The constants used here are given in Appendix A. Some cautionary comments on the determination of humidity are given in Appendix C.

Although the original presentations[23,24] of the BIPM 1981/91 equation showed data restricted to ordinary laboratory conditions, the underlying equations for the compressibility of air and the enhancement factor for water vapor pressure were stated to be valid over very wide ranges of temperature and pressure. The validity of the BIPM equation over the temperature range of interest in this paper has been confirmed by Morris,[25] by direct calculations from the virial coefficients.

4. Refractivity of Moist Air from the BIPM 1981/91 Equation

I have evaluated the refractivity of moist air in two ways, leading to two estimates identified below as n_{prop} and n_{LL}. The first way, which follows Owens,[7] is to assume that the refractivity of each component can simply be added to that of the other components. I therefore multiply the refractivity of dry air by its density relative to that of standard air, and I add the result to the product of the refractivity of pure water vapor and its density relative to the reference density. I do not treat the CO_2 component separately but rather assume[24] that each CO_2 molecule replaces a molecule of O_2, and has the same molecular refractivity. Thus we have

$$n_{\text{prop}} - 1 = (\rho_a/\rho_{axs})(n_{axs} - 1) + (\rho_w/\rho_{ws})(n_{ws} - 1). \quad (5)$$

In Eq. (5) ρ_{axs} is the density of dry air at 15 °C, 101,325 Pa, $x_w = 0$, and a fractional CO_2 content of x_c [found from Eq. (4)], and ρ_{ws} is the density of pure water vapor at 20 °C, 1333 Pa, and $x_w = 1$ [found from Eq. (4)]; these are the conditions to which dispersion equations (1)–(3) apply. Similarly, ρ_a and ρ_w are the densities of the dry air component and the water vapor component of moist air, derived from Eq. (4) for the actual conditions.

The second method for the calculation of the refractivity is to use the additivity of the Lorentz–Lorenz relation[7]:

$$L_i = (n_i^2 - 1)/(n_i^2 + 2), \quad \text{with } i = a(\text{air}), w(\text{water}), \quad (6)$$

whence

$$L = (\rho_a/\rho_{axs})L_a + (\rho_w/\rho_{ws})L_w, \quad (7)$$

$$n_{\text{LL}} = [(1 + 2L)/(1 - L)]^{1/2}. \quad (8)$$

For ordinary laboratory conditions the two methods agree to within a few times 10^{-10} in the refractive index, presumably because of the low concentration of water vapor in typical atmospheres (even saturated air at 20 °C has a partial water vapor pressure of only 23 hPa). Even for saturated air at 60 °C and a total pressure of 80 kPa, the agreement is within 2×10^{-9}.

5. Range of Validity and Predicted Uncertainty of Equations

The BIPM equation for the density of moist air relies on data on the enhancement factor for water vapor and on the compressibility of air. The overall uncertainty (at the 1σ level) in the density was estimated by Giacomo[23] to be $5–13 \times 10^{-5}$, corresponding to an uncertainty in the refractive index of $1.4–3.5 \times 10^{-8}$. The uncertainty of the various measurements of refractive index that were used in the derivation of my equations is a few parts in 10^8. One cannot expect, therefore, an overall uncertainty of better than $2–5 \times 10^{-8}$. In practice the limits to accuracy

Table 1. Phase Refractivity of Dry Air with 450 ppm CO_2 (λ = 633 nm)

Temperature (°C)	Pressure (kPa)	$10^8 (n-1)$ Ref. 3	$10^8 (n-1)$ Eq. (5)
20	80	21458.5	21458.0
20	100	26825.3	26824.4
20	120	32193.0	32191.6
10	100	27776.1	27774.7
30	100	25938.5	25937.2

will be set by the uncertainty of the distribution of the temperature and pressure over the optical path, which, in surveying at least, can only be sampled at a few points. To achieve an uncertainty of 1×10^{-8} in refractive index requires the measurement of the mean temperature to 0.01 °C, which is impracticable. The uncertainty required in the mean pressure is 300 Pa, which is achievable. The requirement for uncertainty of the CO_2 content is less severe, i.e., ~100 ppm. An alternative procedure[26] is to measure the distance at two wavelengths and to assume that the dispersion is almost independent of the temperature and pressure. This results in an uncertainty in distance that is 20–100 times greater than that of the individual measurements, depending on the separation of the two wavelengths; this may be acceptable in surveying over very long distances, but it demands extreme precision over short distances.

6. Comparison with Previous Calculations and with Experiment at λ = 633 nm

Table 1 compares my results for dry air with those calculated by Birch and Downs[4]; the agreement is better than 3×10^{-8}.

The most recent precise absolute measurements of refractive index of moist air are those of Birch and Downs[3,4] and of Hou and Thalmann.[5] Table 2 compares my calculated values with the calculations and measurements of Birch and Downs, which cover a range of conditions likely to occur in a controlled

laboratory. The apparatus used to measure the refractive index has been compared with several other absolute instruments,[6] and the agreement was within a few parts in 10^8.

I found, as expected, excellent agreement between my calculated values and those calculated by Birch and Downs for dry air. However, for moist air there was at first a systematic deviation proportional to the vapor pressure. This deviation[3] was assumed to result from an error in the research of Barrell and Sears, and I corrected it in the same way as did Birch and Downs,[3] by plotting the error against water vapor pressure and deriving a correction to the absolute refractivity of water vapor at the reference conditions (20 °C and 1333 Pa). The result was that the refractivity of water vapor deduced from Barrell and Sears had to be multiplied by 1.022, as indicated in Eq. (3). (This may be interpreted as an error in the estimated vapor pressure of 2.2%.) After I applied this correction to my equations, my results for the refractive index agreed with Birch and Downs to within 1 part in 10^8.

Edlen's original analysis was intended to apply to air in normal laboratory conditions, and various approximations were used to simplify the equations. Birch and Downs were also interested only in these restricted conditions, but it was of interest to compare predictions from their equations for extreme conditions with those developed here, which are based on the BIPM density equation, and therefore, as mentioned above, should be valid over very wide ranges of temperature, pressure, and humidity. I have made such comparisons over the ranges 20–50 °C, 80–120 kPa, and 0–100% relative humidity. Table 3 shows some of these results; the conclusion is that the Birch and Downs predictions would differ from those here by up to 5×10^{-7} in refractive index at high temperatures and humidities. Even at 40 °C and 75% relative humidity (1754 Pa), the difference in refractive index is 1.3×10^{-7}. In the absence of experimental data, neither I nor Birch and Downs

Table 2. Phase Refractivity of Moist Air (λ = 633 nm)

Temperature (°C)	Pressure (Pa)	p_w [a] (Pa)	x_c [b] (ppm)	$10^8 (n-1)$[c,d] Calculated, Ref. 4	$10^8 (n-1)$[c,d] Measured, Ref. 3	$10^8 (n-1)$[c,d] Calculated, Eq. (5)
19.526	102094.8	1065	510	27392.3	27392.3	27392.9
19.517	102096.8	1065	510	27393.7	27394.0	27394.0
19.173	102993.0	641	450	27682.5	27683.4	27682.4
19.173	103006.0	642	440	27686.0	27686.9	27685.8
19.188	102918.8	706	450	27658.7	27659.1	27658.7
19.189	102927.8	708	440	27661.1	27661.4	27660.8
19.532	103603.2	986	600	27800.0	27802.1	27802.0
19.534	103596.2	962	600	27798.7	27800.3	27800.8
19.534	103599.2	951	610	27800.0	27801.8	27802.2

[a] p_w is the vapor pressure of water.
[b] x_c is the CO_2 content.
[c] n is the refractive index.
[d] The refractivity equation given by Birch and Downs[3] [Eq. (5)] has a minor error in the constant term in the denominator. This constant should be such as to make the equation reduce to the standard refractivity at 15 °C and 101,325 Pa; the appropriate value is 96094.69. The correction to the refractivity of water vapor was not reassessed after the correction[4] of the sign of the effect of changing the temperature scale. In column 5 of this table, I have adjusted the constant but have not refitted the water vapor correction.

Table 3. Comparison of Birch and Downs Equation for Phase Refractivity at Extreme Atmospheric Conditions; CO_2 Content = 450 ppm, λ = 633 nm

Temperature (°C)	Pressure (kPa)	Humidity (%)	Vapor Pressure (Pa)	$10^8 (n-1)$ [a] Refs. 3 and 4	$10^8 (n-1)$ [a] Eq. (5)	Difference
20	80	75	1754	21394.8	21394.0	0.8
20	120	75	1754	32129.2	32127.8	1.4
40	80	75	5539	19883.6	19996.5	−12.9
40	120	75	5539	29928.6	29941.8	−13.2
50	80	100	12352	19012.9	19058.4	−45.5
50	120	100	12352	28745.2	28792.4	−47.2

[a]See footnote d of Table 2.

can be confident that the correction to the water vapor density used by Barrell and Sears is adequate in these conditions. (The highest humidity for which experimental data are available is 1065 Pa at 19.5 °C, or ~47%.)

7. Approximations

Although the ready availability of computers lessens the need for approximate formulas, I have investigated the effects of dropping some of the terms involved in the BIPM 1981/91 density calculations. In particular I have tried omitting the terms in Z that involve $(p/T)^2$ and t^2, and setting the enhancement factor f to 1. Because any relative error in the density affects only the refractivity, the effect is reduced by a factor of approximately 3×10^{-4}, so an error of 1×10^{-8} in refractive index results from an error of 3×10^{-5} in density. I have found that the error in the approximation for the refractive index is less than 5×10^{-8} over the ranges $t = 0$–60 °C, $p = 80$–120 kPa, and humidity = 0–100%. In contrast, the quadratic term in the water vapor content cannot be neglected if uncertainties of better than 10^{-7} are required. At low temperatures or humidities below 50%, the error is 1×10^{-8} or smaller. There seems little point in using the approximations, except where the atmospheric conditions can be assumed always to lie in this safe range, or where an error of 2–5×10^{-8} is acceptable.

8. Group Refractive Index

Many electronic distance-measuring instruments use intensity-modulated or pulsed light, and therefore require a knowledge of the group refractive index, rather than the phase index. The group index (n_g) for any component under standard conditions is found by replacement of the corresponding standard phase index (n) by

$$n_g = n + \sigma(\mathrm{d}n/\mathrm{d}\sigma), \qquad (9)$$

with the following results:

$$10^8(n_{gasx} - 1) = \left[\frac{k_1(k_0 + \sigma^2)}{(k_0 - \sigma^2)^2} + \frac{k_3(k_2 + \sigma^2)}{(k_2 - \sigma^2)^2}\right]$$
$$\times [1 + 0.534 \times 10^{-6}(x_c - 450)], \qquad (10)$$

$$10^8(n_{gws} - 1) = \mathrm{cf}(w_0 + 3w_1\sigma^2 + 5w_2\sigma^4 + 7w_3\sigma^6). \qquad (11)$$

(The constants in these equations are given in Appendix A.)

The validity of these equations is therefore directly linked to that of the equations for the phase index. By considering the expression

$$\mathrm{d}n_g/\mathrm{d}n = (\mathrm{d}n_g/\mathrm{d}\sigma)/(\mathrm{d}n/\mathrm{d}\sigma),$$

where n_g is the group index and n is the phase index, we find that for dry air with 450 ppm of CO_2, the uncertainty in n_g is only approximately three times that in n. For water vapor or CO_2 this ratio is in the range 1–3 in the wavelength range covered by this paper, and because the absolute contribution of these components is small, the overall uncertainty should be adequately given by the values for dry air. An alternative analysis, assuming that the uncertainty of each constant in the dispersion equations is one unit in the last digit given, and propagating these uncertainties by the usual statistical rules results in an uncertainty in n_g that is only approximately 1.1 times that in n.

These calculations were based only on the uncertainties of the dispersion equations, which in the case of the phase index are thought to be approximately 1×10^{-9}. It is not always clear in the literature what confidence level is implied, but it is likely that a 1σ level is common. On this basis, the overall uncertainty of the dispersion component of the group index will be less than 3×10^{-9}. Clearly the overall uncertainty of the group index will be dominated by that of the phase index, i.e., by the factors discussed in Sections 5 and 6 of this paper. The group index under practical conditions will be given by application of the method of Section 4 of this paper to the standard group indices given by Eqs. (10) and (11).

9. Conclusion

The equations given here yield the same results as the modified Edlen[4] equations, and they agree with the experimental results to within the published uncertainty of the measurements. However, I believe that for very precise research, and particularly for geodetic surveying, the procedure and equations given here are preferable to the modified Edlen equations, for two reasons. First, they should apply over a much wider range of atmospheric conditions,

because the BIPM density equation is based on data that have been shown to be valid over ranges of at least -40 to $+100$ °C, 80 to 120 kPa, and 0 to 100% relative humidity. Second, they should apply over the wavelength range from approximately 300 nm to 1690 nm, and therefore they should cover the region near 850 nm at which most modern surveying instruments operate.

As discussed above, I also believe that there is little reason to adopt any approximate formulas, given the increasing use of programmable computers, even in field studies. My own calculations have been performed with MATHCAD,[27] but the algorithms have since been translated into FORTRAN. I am willing to supply copies of these programs in electronic form to anyone who is interested. To assist anyone who wishes to program the complicated sequence of calculations, I have included an annotated list of the steps in Appendix B. An outline of the procedure used here has been presented to the geodetic community[28] for possible adoption in place of all versions of the currently used Edlen equations. In controlled laboratory conditions and for visible wavelengths, there is no clear advantage in the new equations; however it should be remembered that the published equations of Birch and Downs[3,4] assume that the CO_2 content of the atmosphere is 450 ppm, and that further correction may be needed as this component varies because of local conditions (e.g., enclosed laboratories[11]) or general atmospheric changes. This correction is allowed for in my program. All current equations assume normal composition of the atmosphere; this is clearly inadequate to deal with enclosed apparatus or industrial environments, where vapors of oils and solvents are known[29–31] to change the refractive index by 1–2 $\times$ 10^{-7}.

There is one other potential contribution to the refractivity that has not been considered in this analysis, nor in most of the other discussions of refractivities in the optical and the near infrared mentioned above. This is the contribution of the wings of infrared absorption lines[16] of water vapor, CO_2, and possibly other gases or vapors (other than those incorporated in the dispersion equations used here). Some research on this topic is being done by members of the IAG Working Group,[32,33] and it may ultimately be necessary to incorporate some corrections, which would be very dependent on wavelength. These would presumably take the form of small corrections added to the component refractivities calculated by the methods of this paper.

Finally, it seems to me that the limiting contribution to the accuracy of distance measurements over long, open paths is set by the difficulty of determining the mean temperature and pressure; the alternative, the two-wavelength method, is highly precise only over distances of many kilometres, and it makes severe demands on the individual measurements. The equations presented in this paper produce results that take into account all known factors (except for atmospheric contaminants such as oil vapors and the effects of absorption lines), embody the latest values of all physical parameters and units, and should be valid for all practical atmospheric conditions and for wavelengths from below 350 nm to above 1300 nm.

Appendix A.

The standard refractivities and the BIPM 1981/91 density equation involve many auxiliary quantities; for clarity in exposition in the main text these are collected here.

The constants involved in the standard phase and group refractivities of dry air (from Ref. 8, amended for changes in temperature scale and CO_2 content) are as follows:

$$k_0 = 238.0185 \ \mu m^{-2}; \qquad k_1 = 5792105 \ \mu m^{-2};$$

$$k_2 = 57.362 \ \mu m^{-2}; \qquad k_3 = 167917 \ \mu m^{-2}.$$

The constants involved in the standard phase and group refractivities of water vapor (from Ref. 7) are as follows:

$$w_0 = 295.235 \ \mu m^{-2}; \qquad w_1 = 2.6422 \ \mu m^{-2},$$

$$w_2 = -0.032380 \ \mu m^{-4}, \qquad w_3 = 0.004028 \ \mu m^{-6}.$$

The constants and equation used in the BIPM 1981/91 density equation are as follows. The saturation vapor pressure of water[24] uses

$$A = 1.2378847 \times 10^{-5} \ K^{-2},$$

$$B = -1.9121316 \times 10^{-2} \ K^{-1},$$

$$C = 33.93711047, \qquad D = -6.3431645 \times 10^3 \ K.$$

The enhancement factor of water vapor[23,24] uses

$$\alpha = 1.00062, \quad \beta = 3.14 \times 10^{-8} \ Pa^{-1},$$

$$\gamma = 5.6 \times 10^{-7} \ °C^{-2}.$$

The compressibility[24] uses

$$Z = 1 - (p/T)[a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t)x_w$$
$$+ (c_0 + c_1 t)x_w{}^2] + (p/T)^2(d + ex_w{}^2), \qquad (12)$$

where

$a_0 = 1.58123 \times 10^{-6} \ K \ Pa^{-1}, \qquad a_1 = -2.9331 \times 10^{-8} \ Pa^{-1}, a_2 = 1.1043 \times 10^{-10} \ K^{-1} \ Pa^{-1},$
$b_0 = 5.707 \times 10^{-6} \ K \ Pa^{-1}, \qquad b_1 = -2.051 \times 10^{-8} \ Pa^{-1},$
$c_0 = 1.9898 \times 10^{-4} \ K \ Pa^{-1}, \qquad c_1 = -2.376 \times 10^{-6} \ Pa^{-1},$
$d = 1.83 \times 10^{-11} \ K^2 \ Pa^{-2}, \qquad e = -0.765 \times 10^{-8} \ K^2 \ Pa^{-2}.$

Appendix B.

To clarify the rather lengthy calculations required to find the phase or group refractive index under any

given conditions, I list here the sequence of operations for the evaluation of the phase index. For the group index, simply replace Eq. (1) and Eq. (2) by Eq. (10), and replace Eq. (3) by Eq. (11).

The input data are wavelength (λ), temperature (t, °C), pressure (p, Pa), partial pressure of water vapor (p_w, Pa), and CO_2 content (x_c, ppm).

1. Find svp, f, x_w, from Appendix A.
2. Find n_{axs} from Eqs. (1) and (2).
3. Find M_a from the text following Eq. (4).
4. Find Z_a (compressibility of dry air) from Eq. (12) by setting $T = 288.15$ K, $p = 101,325$ Pa, and $x_w = 0$.
5. Find Z_w (compressibility of pure water vapor) from Eq. (12) by setting $T = 293.15$ K, $p = 1333$ Pa, and $x_w = 1$.
6. Find the densities of standard air (ρ_{axs}) and of standard water vapor (ρ_{ws}) by using Eq. (4) with the corresponding values of Z.
7. Find the compressibility of moist air (Z) under the experimental conditions (t, p, x_w) from Eq. (12).
8. Find the density of the dry component of the moist air from Eq. (4), using parameters p (total atmospheric pressure), x_w, and Z (note: not Z_0); $\rho_a = pM_a(1 - x_w)/ZRT$.
9. Similarly, find the density of the water vapor component from Eq. (4), using p, x_w, and Z; $\rho_w = pM_w x_w/ZRT$.
10. Find n_{prop} from Eq. (5), or n_{LL} from Eqs. (6)–(8).

Appendix C.

The accurate measurement of humidity is a difficult task. There are doubts about the reliability of the psychrometer coefficients used with Assmann psychrometers, and it is doubtful whether conventional tables of relative humidity allow for the enhancement factor. There are additional problems in using a psychrometer in conditions in which the wet bulb is covered with ice. These matters will have to be considered in assessing the practical, rather than the ultimate, limits to the determination of refractivity.

Because surveying instruments are often certified to operate down to −20 °C, it is necessary to consider the saturation vapor pressure over ice, which differs significantly from that over liquid water. Hyland and Wexler[34] and Jancso *et al.*[35] have presented relevant data and equations. More recently, Marti and Mauersberger[36] reviewed the available data and equations, and they developed a simple two-parameter equation that agrees with the data and with Refs. 32 and 33 to within 1 Pa down to at least −20 °C. This corresponds to an agreement in refractive index to less than 1×10^{-9}. This equation is

$$\log_{10}(\text{svp}) = -2663.5/T + 12.537. \qquad (13)$$

Because the vapor pressure over ice is very small (only 611 Pa at 0 °C), it is adequate to use the BIPM expression for the enhancement factor, or even to set $f = 1$.

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