Erratum: ‘‘Einstein coefficients, cross sections, *f* values, dipole moments, and all that’’ [Am. J. Phys. 50, 982 (1982)]

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$\mathbf{b}$ in the frame. However, it is clear that $\hat{n}$ would transform into a new axis $\hat{n}'$ under a general rotation of the frame according to

$$\hat{n}' = R^{-1}\hat{n}. \tag{14}$$

In view of these observations and the fact that a vector remains invariant under any rotation about the axis along itself and transforms according to a relation similar to Eq. (14) under a general rotation of the frame, $\hat{a}_{\parallel} * \hat{a}_{\perp}$ can be identified as a vector along $\hat{n}$, i.e.,

$$\hat{a}_{\parallel} * \hat{a}_{\perp} = \hat{n}K, \tag{15}$$

where K is a constant number. By appropriately defining the unit of $\hat{a}_{\parallel} * \hat{a}_{\perp}$ in relation with the units of $\hat{a}_{\parallel}$ and $\hat{a}_{\perp}$, K can always be chosen to be unity. Thus

$$\hat{a}_{\parallel} * \hat{a}_{\perp} = \hat{n}. \tag{16}$$

The information that $\hat{n}$ is perpendicular to the plane of $\mathbf{a}$ and $\mathbf{b}$, does not uniquely decide the direction of $\hat{n}$. It is therefore decided conventionally in relation to the directions of $\mathbf{a}$ and $\mathbf{b}$.

From Eqs. (6), (8), and (16) we have

$$\mathbf{a}*\mathbf{b} = ab\cos\theta + \hat{n}ab\sin\theta. \tag{17}$$

Thus the *net* product of two vectors turns out to be the sum[5] of a scalar quantity $ab\cos\theta$ and a vector quantity, $\hat{n}ab\sin\theta$. Such a sum may have some significance as a mathematical entity but does not carry any sense as a physical quantity. However, if both terms are considered separately, they may have some significance in physics too. Therefore, we can separately name and define these quantities as

$$\textit{dot}\ \text{product:}\quad \mathbf{a}\cdot\mathbf{b} = ab\cos\theta, \tag{18}$$

$$\textit{cross}\ \text{product:}\quad \mathbf{a}\times\mathbf{b} = \hat{n}ab\sin\theta. \tag{19}$$

It may be noted that in its definition, the *net* product $\mathbf{a}*\mathbf{b}$ is not much different from the normal algebraic product of two quantities; whatsoever difference exists in its evaluation is a technical necessity. Equations (1) and (2) are, therefore, unique in the sense that they, respectively, account fully for the scalar and vector components of the *net* product [cf. Eq. (17)]. Thus it is clear that we cannot have any alternative of these operations which serve the same purpose. It also becomes clear that the physical observations are not essential for quoting them as the bases for defining these operations the way they are defined; in fact this note provides to these definitions a necessary mathematical background.

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[1]R. Resnick and D. Halliday, *Physics* (Wiley Eastern, New Delhi, 1976), Chap. 2.

[2]G. Arfken, *Mathematical Methods for Physicists* (Academic, New York, 1970), Chap. 1.

[3]See, for example, (i) G. D. Oates in *Handbook of Applied Mathematics* edited by C. E. Pearson (Van Nostand-Reinhold, New York, 1974), Chap. 3; (ii) N. M. Queen, *Vector Analysis* (McGraw-Hill, London, 1967) and several other workers.

[4]Y. S. Jain, Technical Report No. 1/1981, Department of Physics, North-Eastern Hill University, Shillong-3, India (unpublished).

[5]$\mathbf{a}*\mathbf{b} = ab\cos\theta + \hat{n}ab\sin\theta$ should not be sensed as a simple sum of two quantities. This point may be understood by examining the transformation properties of $\mathbf{a}*\mathbf{b}$ particularly under rotation–reflection operations. For example, under inversion/(reflection through the plane of $\mathbf{a}$ and $\mathbf{b}$), $\mathbf{a}*\mathbf{b}$ transforms to $ab\cos\theta - \hat{n}ab\sin\theta$. Using $\mathbf{a} = \hat{\imath}a_x + \hat{\jmath}a_y + \hat{k}a_z$ and $\mathbf{b} = \hat{\imath}b_x + \hat{\jmath}b_y + \hat{k}b_z$ one may easily have $\mathbf{a}*\mathbf{b} = a_xb_x + a_yb_y + a_zb_z + \hat{\imath}(a_yb_z - a_zb_y) + \hat{\jmath}(a_zb_x - a_xb_z) + \hat{k}(a_xb_y - a_yb_x)$. Under reflection through, say $x-y$ plane, it will transform to $\mathbf{a}*\mathbf{b} = a_xb_x + a_yb_y + a_zb_z + \hat{\imath}(a_yb_z - a_zb_y) + \hat{\jmath}(a_zb_x - a_xb_z) - \hat{k}(a_xb_y - a_yb_x)$.

Addendum: "Trouble with the method of images"

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The recent paper by Newcomb[1] on the difficulties involved in summing a series associated with the method of images is very interesting for the detailed discussions it contains. As a supplement to this paper, I wish to bring to the readers' attention several papers and comments which have previously appeared in this Journal.[2–8]

[1]W. A. Newcomb, Am. J. Phys. **50**, 601–607 (1982).

[2]C. Y. Fong and C. Kittel, Am. J. Phys. **35**, 1091–1092 (1967).

[3]J. Pumplin, Am. J. Phys. **37**, 737–739 (1969).

[4]J. J. G. Scanio, Am. J. Phys. **41**, 415–418 (1973).

[5]B. G. Dick, Am. J. Phys. **41**, 1289–1290 (1973).

[6]M. Zahn, Am. J. Phys. **44**, 1132–1134 (1976).

[7]J. Pleines and S. Mahajan, Am. J. Phys. **45**, 868–869 (1977).

[8]G. Simon, Am. J. Phys. **47**, 566 (1979).

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The numerator of the first factor in Eq. (25) should be $\gamma_{\rm cl}/2\pi$. Consequently, the numerical values of $\sigma_a(\omega = \omega_{21})$ and $g(\omega_{21})$ in Sec. VIII should each be reduced by a factor of 2. In the caption for Table I, the correct equations are $B_{12} = (g_2/g_1)B_{21}$, $f_{12} = -(g_2/g_1)f_{21}$. I thank A. E. Siegman for pointing out the factor of 2 error.