Figure 4 Pulse-induced current as a function of the pulse length $\Delta t$. The data correspond to the cross-section of Fig. 3a at $Q_0/e = 0.51$. Inset, Josephson energy $E_J$ versus the magnetic flux $\phi$ penetrating through the loop. $E_J$ was estimated by two independent methods. One was from the period of the coherent oscillation $T_{coh}$ as $h/T_{coh}$. The other was from the gap energy observed in microwave spectroscopy $E_{g}$. The solid line shows a fitting curve with $E_g(e=0) = 84 \mu$eV assuming cosine $\phi$-dependence of $E_J$.

Figure 4 shows the pulse-induced current at $Q_0/e = 0.51$ as a function of $\Delta t$, showing that the coherent oscillation can be observed in the time domain and that we can control the quantum state through an arbitrary pulse length $\Delta t$. The oscillation amplitude was smaller than that simply expected from $2e$ per pulse, $2\phi/T = 20 \text{pA}$. The finite rise and fall times of the pulse might explain this deviation. We recall that in the limit of long rise and fall times (the adiabatic limit), there would be no transition probability $\sin^2 \alpha_t$ as described in the main text. For the realistic rise and fall times of the pulse we assumed in the simulation above, for example, the amplitude of the oscillations in $\Delta P(E)$ at $Q_0/e = 0.51$ is reduced to $\sim 0.4$, by which the current signal would be decreased. Moreover, the finite repetition time (not much longer than $1/T_{coh}$) could also reduce the signal due to the incomplete relaxation of $\sin^2 \alpha_t$ at each pulse.

To further confirm that the observed oscillation was coherent oscillation due to Josephson coupling, we estimated the Josephson energy $E_J$ from the oscillation period $T_{coh}$ as $E_J = h/T_{coh}$ and investigated its magnetic-field dependence (filled circles in Fig. 4 inset). We also measured $E_J$ in the frequency domain through microwave spectroscopy of the energy-level splitting $\Delta P(E)$ (open squares in Fig. 4 inset). The two sets of data agreed very well, and fitted the expected cosine curve.

For future application as quantum computing devices, a crucial parameter is the decoherence time. The main decoherence source in a single-Cooper-pair box is thought to be spontaneous photon emission to the electromagnetic environment $\gamma_{J}$, and the decoherence time could exceed $\sim 1 \mu$s when a probe junction is used, as in our set-up, the ‘detection’ with quasiparticle tunnelling through the probe junction would be the main source of decoherence. So far, we have observed oscillation up to $\Delta t \sim 2 \text{ns}$, although low-frequency background charge fluctuation degraded the direct current signal and made it difficult to determine the envelope of the decay. A more detailed study of the decoherence time would provide important information for designing solid-state quantum circuits using superconducting single-Cooper-pair boxes.
between ASW and crystalline ice and determined that thermodynamic continuity with normal liquid water is possible\(^1,2\). We have also shown that long-range molecular translational diffusion characteristic of liquid-like behaviour occurs before crystallization\(^3,4\). Estimates of the liquid diffusion coefficients yield a diffusivity roughly a million times greater than that of crystalline ice\(^5\).

We employ molecular beams of H\(_2\)\(^{16}\)O and H\(_2\)\(^{18}\)O with high spatial resolution to create nanoscale ASW films (<5,000 Å thick). Temperature-programmed desorption (TPD) is used to measure quantitatively the desorption kinetics, and thereby reveal the extent of mixing between the H\(_2\)\(^{16}\)O and H\(_2\)\(^{18}\)O layered interfaces\(^17,21\).

Quantitative measurements of the desorption rates from the amorphous and crystalline phases of H\(_2\)O and D\(_2\)O have been used to quantify the crystallization kinetics which can be described by a classic nucleation and growth mechanism\(^17,20\). The conversion to the thermodynamically stable crystalline phase results in a concomitant decrease in the vapour pressure (desorption rate) and the irreversible amorphous to crystalline transformation\(^14,16,20,21\) appears as a ‘bump’ in the 150–160 K temperature range of the TPD spectrum.

We quantify the temperature-dependent diffusivity using a mathematical model that couples our previous mean-field description of the desorption/crystallization kinetics\(^20\) to a one-dimensional representation of the diffusive transport between layers. The effective diffusivity is a linear combination of the amorphous and crystalline diffusion coefficients weighted by their respective mole fractions. We assume an Arrhenius form for the temperature dependence of the diffusion coefficients, and vary the amorphous diffusion parameters to achieve a fit to the experimental data.

Figure 1 shows TPD spectra and model simulations for two different total thicknesses of ASW. As shown in Fig. 1a, the onset of isotopic mixing is abrupt and is concomitant with the phase transformation of ASW into crystalline ice. If the isotopically tailored layers did not exhibit diffusive intermixing, all of the H\(_2\)\(^{18}\)O molecules would desorb before the onset of H\(_2\)\(^{16}\)O desorption. This would be expected if the ASW phase had the diffusivity of...
crystalline ice. The simulation results (solid lines) are in excellent agreement with the experiment. The four illustrations in the bottom of Fig. 1a show the vertical spatial distribution of the isotopes obtained from the simulation. At 144 K the two isotopes are vertically separated as deposited, but by 153 K significant intermixing has occurred. At 159 K the layers have completely mixed and the film has crystallized.

Figure 1b shows the experimental TPD spectra and the model simulation for a much thicker film. These spectra reveal that only a limited amount of mixing has occurred before complete crystallization. The vertical spatial distributions of the isotopes, illustrated in the bottom of the figure, show that mixing occurs in concert with the phase transition but that at 159 K the film has not mixed completely. Above 159 K the film is completely crystallized, and as such, the diffusive motion is 'frozen out'. As the film thickness increases, the films show departures from complete mixing because the effective mixing zone is limited by the time it takes for the film to crystallize. We have performed similar experiments for a variety of films, and find that the model simulation with a single set of amorphous diffusion parameters is in excellent agreement with the experimental data for thicknesses from 30 to 200 monolayers (ML) irrespective of which isotope is on top. Although not shown here, experiments using H/D labelled water reveal complete isotopic scrambling within the intermixed region, thereby demonstrating intimate chemical contact between the species.

The temperature dependence of the diffusivity obtained from the simulation is shown in Fig. 2. In Fig. 2a the diffusivity (solid line) shows an initial exponential rise with temperature but reaches a maximum near 157 K and then drops rapidly. As the fraction crystallized increases from 0 to 1 (dashed line), the diffusivity transforms from completely amorphous diffusion to completely crystalline diffusion. As the diffusion in the amorphous liquid phase is much easier than in the crystalline solid phase, the effective diffusivity shows a maximum when plotted as a function of temperature. The diffusivity is shown in an Arrhenius plot in Fig. 2b. The low-temperature diffusivity data show that the amorphous diffusion has a relatively high activation energy. An Arrhenius fit (dashed line) to the diffusivity over the temperature range 147–157 K yields an apparent activation energy of 170 ± 40 kJ. The large uncertainty in the activation energy is due to the insensitivity of the simulations to the slope of the Arrhenius fit over the narrow temperature range before the onset of crystallization. Nonetheless, the simulations are extremely sensitive to ±50% variations in the absolute value of the amorphous diffusivity over this narrow temperature range. Above 157 K, the diffusivity drops rapidly owing to crystallization. Diffusion in crystalline ice in this temperature range (up to 170 K) is extremely small, and nearly identical simulation results are obtained when the value of crystalline-diffusion is set to zero. Near 160 K the diffusivity of ASW reaches (~10^-12 cm^2 s^-1), a value 10^6 times smaller than the diffusivity of normal liquid water at room temperature. Such a diffusivity would be nearly impossible to observe with a macroscopic sample—a 1-cm-thick film would require ~10^9 years to mix completely. The use of nanoscale films enables these small diffusivities to be determined quantitatively.

Figure 3 shows the temperature dependence of the diffusivity for ASW, liquid water, supercooled liquid water, and crystalline ice. An Arrhenius extrapolation of the liquid and supercooled liquid data does not agree with our ASW diffusivity results; our results show a much stronger temperature dependence, with an apparent activation energy of ~170 kJ mol^-1. It is known that glass-forming liquids show markedly non-Arrhenius behaviour as they are supercooled below their freezing point. The temperature dependence of this non-Arrhenius behaviour is often well represented by the empirical Vogel–Fulcher–Tammann (VFT) equation,

\[ \frac{1}{D} = \frac{1}{D_0} + \frac{\gamma}{(T - T_g)} \]  

where \( D \) is the temperature dependent diffusivity, \( T \) is the temperature, and \( D_0, \gamma, T_g \) are fit parameters. The solid line labelled VFT in Fig. 3 is the result of fitting all of the diffusivity data to the VFT equation.

It has been proposed that there is a thermodynamic singularity in the water phase diagram at 228 K. This proposal was partly based on the observation that many of the physical properties of supercooled water could be described by a power-law equation of the form,

\[ A = A_0 (T/T_g - 1) \gamma \]  

where \( A \) is the property of interest and \( A_0, \gamma, T_g \) are fit parameters. One interpretation of this singularity is given by the “stability limit conjecture”, which predicts that normal liquid water cannot exist below Tg due to a retraction spinodal. Another interpretation is that the apparent singularity is due to the existence of a second critical point in the water phase diagram. The power-law prediction for the liquid and supercooled-liquid diffusivity data is plotted in Fig. 3. Over the temperature range 250–500 K, a comparison of the fits does not provide unambiguous evidence in support of either the VFT or power-law equations. On an expanded scale (Fig. 3 inset), the VFT equation deviates from the liquid data below 250 K by about 25% and the power law gives a better fit. However, assuming that the diffusivities from the two temperature regions can be connected, then the VFT equation describes the entire set of data spanning a range of 10^10 reasonably well (within a ±25% deviation). We note that the power-law equation, or any equation with a temperature singularity at 228 K, cannot fit both the ASW and the liquid diffusivity data.

The open question is whether there is a continuity between the normal supercooled-liquid diffusivity and the liquid-like diffusivity...
near 150 K. The ASW diffusivity data provide for an interpretation that does not require either a singularity at 228 K (refs 3, 28) or a new distinct phase of liquid water16,17. The continuity interpretation that does not require either a singularity at 228 K (refs 3, 28, 29) or a thermodynamic instability is required to explain the anomalous behaviour of the transport properties near the proposed temperature singularity28—although these simulations use the SPC-E water potential which is known to represent inadequately some aspects of the normal liquid behaviour.

Taken together, the above results, combined with our previous demonstration of a thermodynamic continuity5, suggest that at the calorimetric glass transition temperature of 136 K (ref. 31) the amorphous solid melts into a deeply supercooled metastable extension of normal liquid water before crystallizing near 160 K. This interpretation does not require the existence of a temperature singularity near 228 K (ref. 2) at low pressure (<0.1 MPa), and is consistent with (but is not proof of) the existence of a second critical point near 220 K and 0.1 GPa (refs 8, 12). Although our ASW diffusivity data provide support for a continuity between ASW and liquid water at low pressure, an ambiguous resolution of the continuity conundrum must await further experiments in the unexplored temperature regime from 160 to 240 K. Such experiments will be difficult because of the rapid crystallization of supercooled liquid water below 230 K, and of ASW above 160 K.

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