TABLE II. α-particle and triton binding energies for equal central and tensor force ranges.

$r_c=r_t=1.36\times10^{-13}$ cm, $\gamma=1.80$, $V_0\simeq25.56$ Mev, $g=-0.191$	
$E=7.0$ Mev[a]	$E=6.7$ Mev[b,c]

[a] α-particle. [b] Triton. [c] See reference 3.

force ranges, however, very little binding energy is obtained for the α-particle, so that the result of Hu and Hsu[3] for the triton for a particular set of nuclear constants seems incompatible with the α-particle binding energy. The values are given in Table II.

A detailed account of these calculations will be published elsewhere.

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[1] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951).
[2] R. L. Pease and H. Feshbach, Phys. Rev. **81**, 142 (1951).
[3] T. Hu and K. Hsu, Proc. Roy. Soc. (London) **A204**, 476 (1951).
[4] J. Irving, Phil. Mag. **42**, 338 (1951).

The Specific Alpha-Activity of U^{234}*

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THE half-life of uranium 234 has been the subject of several recent experimental investigations. Nier[1] obtained a value of $(2.69\pm0.27)\times10^5$ years. Chamberlain, Williams, and Yuster[2] reported values of $(2.29\pm0.14)\times10^5$ years and $(2.35\pm0.14)\times10^5$ years; Knight, Goldin, Macklin, and Macklin[3] derived a value of $(2.69\pm0.04)\times10^5$ years; and the author[4] reported a value of $(2.522\pm0.008)\times10^5$ years. Because of the wide range of values and also because of the general interest in the exact value of the uranium 234 half-life, which enters into the quantitative determination of uranium 234 by alpha-counting, it seemed advisable to re-determine the specific alpha-activity of uranium 234. A new approach had been made available through the production of significant quantities of uranium containing 94.70 ± 0.08 weight percent uranium 234, 4.02 ± 0.03 percent uranium 235, and 1.28 ± 0.03 percent uranium 238 as determined by mass spectrometry. Spectrographic analysis established that the chemical purity of the oxide was better than 99.95 percent.

Samples of the oxide were weighed to ±0.1 percent, dissolved in concentrated nitric acid, and diluted to volume. Volume aliquots of these solutions were transferred to an electroplating cell. Uranium depleted in the 234 and 235 isotopes (U^{238}) was added to the plating cell to act as a diluent and the uranium was electrodeposited quantitatively onto an electropolished nickel disk with an area of 23 cm^2. Various sized aliquots of the diluent (U^{238}) were used in order to obtain varying thicknesses of the uranium oxide film. Uniform films resulted, varying in thickness from 0.008 to 0.33 mg/cm^2. Similar films were made of the diluent, and their counting rate was subtracted from the total counting rate, leaving the counting rate for the uranium oxide enriched in the 234 isotope. The alpha counting rate was corrected for the diluent absorption of alpha-particles by extrapolating to zero film weight thickness. After this correction was made, the counting rate of the uranium enriched in 234 was calculated to be $(0.6457\pm0.0013)\times10^7$ counts per minute per milligram.

The correction for the reflection or backscattering of the alpha-particles by the nickel was determined by electroplating enriched uranium onto nickel, platinum, and gold disks, which were polished to a mirror finish, and extrapolating the counting rates to zero atomic number for the reflector. The percentage backscattering of alpha particles was found to be as follows: gold 3.36 ± 0.36, platinum 3.31 ± 0.32, and nickel 1.19 ± 0.08. These backscattering values are within the range of those given by Crawford.[5] Since the parallel plate ionization chamber detects the alpha-particles within an angle of 180°, a counting efficiency of 50.595 percent was used to convert the counts to $(1.2762\pm0.0038)\times10^7$ disintegrations per minute per milligram of uranium. Counting corrections for the uranium 235 and 238 content were insignificant. Thus, the specific alpha-activity of uranium 234 is $(1.347\pm0.004)\times10^7$ disintegrations and the half-life calculated from it is $(2.520\pm0.008)\times10^5$ years. This half-life value confirms our recently published value of $(2.522\pm0.008)\times10^5$ years, which is compared with the other published values in the introduction of this paper.

* This paper is based on work performed for the AEC by Carbide and Carbon Chemicals Company, a Division of Union Carbide and Carbon Corporation.

[1] A. O. Nier, Phys. Rev. **55**, 150 (1939).
[2] Chamberlain, Williams, and Yuster, Phys. Rev. **70**, 580 (1946).
[3] Knight, Goldin, Macklin, and Macklin, Phys. Rev. **76**, 336 (1949).
[4] C. A. Kienberger, Phys. Rev. **76**, 1561 (1949).
[5] Seaborg, Katz, and Manning, *The Transuranium Elements: Research Papers* (McGraw-Hill Book Company, Inc., New York, 1949), Paper No. 16-55, National Nuclear Energy Series, Plutonium Project Record, Vol. 14B, Div. IV.

Condensation Phenomenon of an Ideal Einstein-Bose Gas. II

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AFTER publication of our first note[1] on this subject, our attention has been called to a Letter to the Editor[2] in which the density distribution of the lowest eigenstate has already been investigated quantitatively. Lamb and Nordsieck[2] obtained with the aid of the WKB method a result which is qualitatively and semi-quantitatively in agreement with reference 1. Since this Letter to the Editor[2] has also been overlooked in two other recent investigations[3] and since, furthermore, all previous results were stated without proof, we may perhaps be justified in saying a few words about our more complete treatment.

One can easily show that the eigen function of the vertical motion of a particle in a homogenous gravitational field is rigorously given by

$$\psi(z)=\zeta^{\frac{1}{2}}[c_1J_{\frac{1}{3}}(\tfrac{2}{3}\zeta^{\frac{3}{2}})+c_2J_{-\frac{1}{3}}(\tfrac{2}{3}\zeta^{\frac{3}{2}})]. \tag{1}$$

In (1), $J_{\pm\frac{1}{3}}$ denote the Bessel functions of order $\pm\frac{1}{3}$, respectively, and ζ is defined by

$$\zeta=(8\pi^2m^2g/h^2)^{\frac{1}{3}}[(E/mg)-z]. \tag{2}$$

The eigenvalues under the boundary conditions $\chi=0$ for $z=0$, a are given by the roots of the equation

$$\frac{J_{\frac{1}{3}}(\frac{2}{3}\zeta_1)}{J_{-\frac{1}{3}}(\frac{2}{3}\zeta_1)}=\frac{J_{\frac{1}{3}}(\frac{2}{3}\zeta_2)}{J_{-\frac{1}{3}}(\frac{2}{3}\zeta_2)} \tag{3}$$

$$\zeta_1=(8\pi^2m^2g/h^2)^{\frac{1}{3}}E/mg \tag{4a}$$

$$\zeta_2=(8\pi^2m^2g/h^2)^{\frac{1}{3}}(E/mg-a). \tag{4b}$$

These roots can easily be determined, since it turns out that asymptotic expressions for the Bessel functions may be used. One thus obtains for the low eigenvalues the expression

$$E_n=mg(3\pi/2)^{\frac{2}{3}}(h^2/8\pi^2m^2g)^{\frac{1}{3}}(n-\tfrac{1}{4})^{\frac{2}{3}}. \tag{5}$$

The density distribution agrees with that given by Lamb and Nordsieck apart from a slight change in the numerical factor.

For the statistical problem it is, on the other hand, quite sufficient to use an extremely simplified variational method which gives all necessary information and allows us to visualize the physical conditions. We use as a trial eigenfunction for the lowest energy state the function

$$\psi_1(z)=\sin(\pi z/l_0) \quad \text{for} \quad 0\leqq z\leqq l_0 \tag{6a}$$

$$\psi_1(z)=0 \quad \text{for} \quad l_0\leqq z\leqq a, \tag{6b}$$

which satisfies all boundary conditions and leads for the eigenvalue to the expression

$$E_1=\tfrac{1}{2}mgl_0+(h^2/8ml_0^2). \tag{7}$$

The lowest eigenvalue is certainly smaller than the minimum of (7) taken with respect to l_0. We thus obtain

$$l_0=(\hbar^2/2m^2g)^{\frac{1}{3}} \tag{8a}$$

$$(E_1/mg)=\tfrac{3}{4}(\hbar^2/2m^2g)^{\frac{1}{3}}. \tag{8b}$$

The agreement with (5) is obviously very satisfactory. Since both kinetic and potential energy are positive, it is clear that the density must be vanishingly small for values of z sizeably in excess of $l_0/2$. This leads for He to a film thickness of approximately $4\cdot10^{-4}$ cm.

We would like to maintain the view expressed in our first letter that the conditions analyzed by Lamb and Nordsieck and by ourselves clearly indicate that the behavior of an ideal gas is not even approximately indicative of that of He II; the difference between He^4 and He^3, if not mainly a mass effect, may still be caused by the influence of the $E-B$ statistics on the behavior of a liquid. The fact that ordinary He^4 condenses with a density about one thousand times smaller than that of an ideal condensed gas is, in our opinion, convincing evidence that the inter-atomic forces are the determining factor in the distribution law.

[1] O. Halpern, Phys. Rev. **86**, 126 (1952).
[2] W. Lamb and A. Nordsieck, Phys. Rev. **59**, 677 (1941).
[3] R. Becker, Z. Physik **128**, 120 (1950); G. Leibfried, Z. Physik **128**, 133 (1950).

The Influence of Charge Independence of Nuclear Forces on Electromagnetic Transitions

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In a recent paper Trainor[1] has considered the effect of symmetry on the nuclear dipole radiation. The result of his calculations is that the symmetry properties of nuclear states give rise to certain selection rules for electric dipole radiation. We want to draw attention to the fact that this result is much more general and applies not only to electric dipole radiation but also to radiation of any multipolarity. Secondly, we will show that the selection rule obtained is not restricted to the case (considered by Trainor) when the nuclear forces are pure Wigner- and Majorana-type forces, but holds provided only that the forces are charge-independent. This seems a reasonable approximation and is consistent, for example, with the most recent spectroscopic evidence on the excited nuclear states.[2]

The interaction between the electromagnetic field and the nucleus can be represented, in a nonrelativistic approximation by the Hamiltonian

$$H=(e/mc)\Sigma_i\tfrac{1}{2}p_i\cdot A(x_i)(1-\tau_z^{(i)})+\Sigma_i\{\tfrac{1}{2}\mu_n(1+\tau_z^{(i)})+\tfrac{1}{2}\mu_p(1-\tau_z^{(i)})\}\sigma^{(i)}\cdot\mathrm{curl}A(x_i), \tag{1}$$

where p_i is the momentum of the ith nucleon, $A(x_i)$ the vector potential of the electromagnetic field at the position x_i of the nucleon, μ_n and μ_p the neutron and proton magnetic moments, $\sigma^{(i)}$ the Pauli spin matrix, and $\tau_z^{(i)}$ the z-component of the isotopic spin matrix having eigenvalues $+1$ for neutrons and -1 for protons.

If the forces are charge-independent, the total isotopic spin

$$T=\Sigma_i T^{(i)}$$

is a constant of the motion, so that each nuclear stationary state can be assigned a definite eigenvalue $T(T+1)$ of T^2.

Consider now the matrix element

$$\langle\alpha'J'T'|H|\alpha JT\rangle \tag{2}$$

between two states of angular momentum J and J', isotopic spin T, T' (α, α' indicate any other quantum number necessary to specify the state). It is easy to see that the matrix element (2) vanishes unless

$$T-T'=0,\pm1. \tag{3}$$

This follows immediately if we note that H can be written as

$$H=H_0+K_z=H_0+\Sigma_i f_i\tau_z^{(i)}, \tag{4}$$

where H_0 represents the part of H independent of the isotopic spin, and f_i is the factor multiplying $\tau_z^{(i)}$. In isotopic spin space H_0 transforms like a scalar and K_z like the third component of a vector. The matrix elements of H_0 vanish unless $T=T'$. As for K_z, using the same considerations developed by Condon and Shortley[3] for a general vector operator one deduces immediately Eq. (3).

Equation (3) represents a generalization of the result obtained by Trainor[1] for the case of dipole radiation and for nuclei containing an equal number of protons and neutrons. If we restrict ourselves to a consideration of electric dipole radiation, it is well known that the scalar term H_0 [Eq. (4)] gives no contribution to the matrix element since it represents the contribution of a system of particles each with a charge $e/2$. For the second term of Eq. (4) another restriction is immediately obtained considering the dependence of the matrix element (2) on the z-component of the isotopic spin: With the notations of Condon and Shortley one has[4]

$$\langle\alpha JTT_z|K_z|\alpha'J'TT_z\rangle=\langle\alpha T\|K_z\|\alpha'T\rangle T_z.$$

If $T=0$ the right-hand side vanishes, so we can conclude that an electric dipole transition cannot take place between two states with $T=0$. Of course, the transition can still occur as magnetic dipole or as an electric transition of higher multipolarity, since in such a case the matrix element of H_0 does not vanish.

The validity of the selection rule (3) is not restricted to the validity of Wigner's model of nuclear structure; in particular it is still true also when a strong spin-orbit interaction is present, in which case the symmetry of the space part of the wave function is no longer a constant of the motion.

On the other hand, since the Coulomb energy

$$H_0=(e^2/8)\sum_{i<j}(1-\tau_z^{(i)})(1-\tau_z^{(j)})/r_{ij}$$

does not commute with T, the isotopic spin is no longer a constant of the motion, and Eq. (3) will not be strictly satisfied even if the specific nuclear forces are strictly charge-independent. The extent to which Coulomb forces destroy the validity of the selection rule (3) is under investigation and will be reported later.

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[1] L. E. H. Trainor, Phys. Rev. **85**, 962 (1952).
[2] S. Baskin and H. T. Richards, Phys. Rev. **84**, 1124 (1951); W. A. Fowler and T. Lauritsen, Phys. Rev. **82**, 197 (1951); J. D. Seagrave, Phys. Rev. **85**, 197 (1952).
[3] E. U. Condon and G H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1951) p. 59.
[4] Reference 3, p. 63.

Are Direct Nucleon-Lepton Interactions Charge-Independent?

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VOTRUBA[1] has recently suggested that direct nucleon-lepton interactions might be charge-independent. If this were so, the coefficients of $H_{PNe\nu}$, $H_{NPe\nu}$, $\frac{1}{4}H_{PPee}$, $-\frac{1}{4}H_{NNee}$, $-\frac{1}{4}H_{PP\nu\nu}$, and $\frac{1}{4}H_{NN\nu\nu}$ in the Hamiltonian should be equal, if H_{ijkl} means the space integral of any of the invariants $\Sigma_n\bar{\psi}_i\omega_n\psi_j\bar{\psi}_k\omega^n\psi_l$, where we assume the neutrino ν to be different from the antineutrino.[2]

In this connection it is interesting to remark that the phenomenologic direct-interaction terms with $\omega_n=\gamma_\lambda$ (vector coupling; $H_{ijkl}=-\psi_i^\dagger\alpha_\lambda\psi_j\psi_k^\dagger\alpha^\lambda\psi_l$) do not satisfy this condition. In fact, the repulsive interaction $\frac{1}{4}H_{PPee}$ describing phenomenologically the Lamb shift[3] has a coefficient $(4.91\pm0.01)\times10^{-41}$ cm^3 erg, while $(-\frac{1}{4}H_{NNee})$ describing phenomenologically the attractive electron-neutron interaction[4-6] has a coefficient (2.8