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A System with Infinitely Degenerate Bound States

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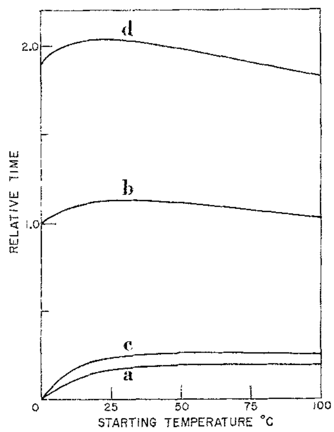

FIG. 1. Cooling of unit mass of water by evaporation. Curves a and c give the time to cool to the freezing point, and b and d the time to finish freezing. For a and b the partial pressure p_a, of Eqs. (1) and (3) is zero; for c and d it corresponds to the vapor pressure of liquid water at −10°C.

and for p_a equal to the vapor pressure of liquid water at −10°C, which equals that of ice at −8.9°C. All curves, both for the time to cool to the freezing point and for the finish of freezing, show maxima. The maxima are more pronounced in the time to finish freezing. The mass lost when cooling is by evaporation is not negligible. Water cooling from 100°C has lost 16% of its mass by 0°C, and loses a further 12% on freezing, for a total loss of 26%.

An experiment in which the cooling can be followed when the water is hot will take an inconveniently long time to reach freezing. For example, water cools from 100° to 50°C in less than 1/10 the time taken to reach 0°C. Measurements are most convenient when the water is 50° to 80°C. Accordingly, an experiment was made with water in two-liter, wide-mouthed Dewar flasks outdoors in a light breeze when the temperature was −6.5°C. The hotter flask started with 1550 g at 88°C, and the cooler with the same mass at 61°C. Temperatures were plotted as a function of time. When the hotter had cooled to 78°C, and the other to 56°C, the ratio of slopes on the graph was 2.1, in agreement with the ratio from Eq. (1) of 2.0, but not in agreement with the ratio for Newtonian cooling of 1.4. Thus, evaporation controls the cooling in this temperature range to the precision of this experiment. Furthermore, when the warmer water had cooled to 39°C, the mass was 1440 g compared to a prediction from the computer program of 1430 g. Again, agreement with the hypothesis of cooling by evaporation alone is to the precision of the measurements.

Thus, experiment and theory agree that hot water freezes faster than cold for sufficiently high starting temperatures, if the cooling is by evaporation. Cooling in a wooden pail or barrel is mostly by evaporation. In fact, if the density of water is considered along with Fig. 1, it is seen that a volume of water starting at 100°C would finish freezing in 90% of the time taken by an equal volume starting at room temperature. The folklore on this matter may well have started a century or more ago when wooden pails were usual. Considerable heat is transferred through the sides of metal pails, so the belief is unlikely to have started from correct observations after metal pails became common.

[1] All data is taken from *Lange's Handbook of Chemistry* (Handbook Publishers Inc., Sandusky, Ohio, 1946), 6th ed.

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As is well known, the Hamiltonian for a charged spinless particle in a constant magnetic field, B, directed towards the z axis is,

$$H=p^2/2M+eBL_z/2Mc+e^2B^2(x^2+y^2)/8Mc^2+V(x,y,z). \quad (1)$$

Choosing $V(x,y,z)=(k_xx^2+k_yy^2+k_zz^2)/2$, Eq. (1) becomes

$$H=p^2/2M+eBL_z/2Mc+\tfrac{1}{2}[(k_x+e^2B^2/4Mc^2)x^2 + (k_y+e^2B^2/4Mc^2)y^2+k_zz^2]. \quad (2)$$

Assuming that $k_x=k_y<k_z=k$, and demanding that

$$k_x+e^2B^2/4Mc^2=k, \quad (3)$$

then Eq. (2) becomes

$$H=p^2/2M+kr^2/2+eBL_z/2Mc. \quad (4)$$

The eigenfunctions of H are $\psi_{n,l,m}=R_{n,l}(r)Y_{l,m}(\theta,\varphi)$, and the eigenvalues

$$E_{n,l,m}=\hbar\omega(2n-l-1/2)+eB\hbar m/2Mc, \quad (5)$$

where

$$\omega=(k/M)^{1/2}, \qquad n=1,2,\cdots,^1 \qquad l=0,1,2,\cdots,n-1,$$

and

$$m=-l,l+1,\cdots,l. \quad (6)$$

Choosing B so that $eB/2Mc=\omega$, it follows that $k=e^2B^2/4Mc^2$, and from Eq. (3), $k_x=k_y=0$.
The eigenvalues, cf. Eq. (5), become

$$E_{n,l,m}=\hbar\omega(2n-l+m-1/2). \quad (7)$$

Setting

$$T=2n-l+m, \quad (8)$$

it follows that the eigenvalues are equally spaced with $T=2,3,\cdots$. For any given T it follows from Eqs. (6) and (8), that $T/2\le n<\infty$ and $n-T/2\le l\le n-1; -l\le m\le l$, i.e., the corresponding level is infinitely degenerate. The possibility of deriving a corresponding noncompact invariance group seems interesting.

[1] It should be mentioned that the n defined in Eq. (5) is the radial quantum number and not the sum of the rectangular quantum numbers $n_{x'}n_{y'}n_z$.