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Comments on "Agreement between Classical and Quantum Mechanical Solutions for a Linear Potential inside a One-Dimensional Infinite Potential Well"

Noble M. Johnson and John N. Churchill

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there is no net movement of the gas. The student should be able to show that the above equation is wrong, especially as the correct identity,

$$\langle u_x^2 \rangle_{\mathrm{av}} = \langle v_x^2 \rangle_{\mathrm{av}} + u^2,$$

can be derived from Glover's Eq. (6). If the student avoids the above pitfall he will obtain a temperature rise of about $\frac{1}{2}$°K; a temperature rise perhaps not at variance with his experience?

The author then presents an hypothesis that as $v_{x\ \mathrm{rms}}$ increases owing to a wind in the x direction there is a "proportionate"—presumably he means "consequent"—decrease in $v_{y\ \mathrm{rms}}$ and $v_{z\ \mathrm{rms}}$ to maintain the kinetic energy value "approximately constant." As equality signs are used throughout the rest of the argument we can only take the word "approximately" as being superfluous. If all the gas molecules have the same mean velocity u, then the relative velocities of all the molecules are unchanged and equipartition of energy is preserved. If there is transfer of internal energy to energy of mass motion, the time required for restoration of thermal equilibrium is of the order 10^{-9} sec for air at atmospheric pressure.[2] The author's hypothesis implies an equilibrium state of the gas where equipartition of internal energy does not hold. Since the total kinetic energy of the gas molecules is assumed to be the same in the wind state as it is in the calm state, the perceptive student will conclude that no external work needs to be done on a windless atmosphere to set it in ordered motion—just a little internal rearrangement, and a neglect of the Second Law.

Our thanks are due to Mr. R. M. Sillitto for discussions during the preparation of this note.

F. H. Barnes
T. Robertson
Department of Natural Philosophy
University of Edinburgh
Edinburgh, Scotland

[1] F. Glover, Amer. J. Phys. **36**, 811 (1968).
[2] L. Rosenhead, *Laminar Boundary Layers* (Oxford University Press, Oxford, 1963), p. 7.

Reply to Letter of Dr. Barnes and Dr. Robertson

(Received 7 March 1969)

The critical student reading in this journal about the "Bernoulli Effect and Kinetic Theory"[1] will indeed perceive that a recent author has goofed, and goofed badly, as Barnes and Robertson, and others[2] have pointed out. That author is presently sweating out his own 15°C temperature rise. Equation (6) of that paper gives the correct method of velocity addition.

Taking as superfluous the word "approximately," as done by Barnes and Robertson, leads to results both wondrous and strange, and the author pleads that his approximately be not disturbed. The requirement that "external work needs to be done on a windless atmosphere to set it in ordered motion," the increase in pressure in the wind direction, and the pressure decrease transverse to this, all seem to be incontrovertible. But just how these ingredients are to be stirred into the brew so that a consistent kinetic theory picture comes out of the cauldron is perhaps a question that is still up for grabs. Further criticisms are *still* invited.

Francisco Glover, S. J.
Ateneo de Manila University
Manila, Philippines

[1] F. Glover, Amer. J. Phys. **36**, 811 (1968).
[2] Prof. Edgar Pearlstein, University of Nebraska (private communication).

Comments on "Agreement between Classical and Quantum Mechanical Solutions for a Linear Potential inside a One-Dimensional Infinite Potential Well"

(Received 16 June 1969)

The quantum mechanical problem of a particle in a potential well defined by the potential energy function

$$\begin{aligned} V(X) &= CX; \qquad 0 \le X \le X_0 \\ &= \infty, \qquad \text{elsewhere,} \end{aligned} \tag{1}$$

where C is a constant, was recently treated by Dymski[1] for the special case in which the total energy of the particle is equal to the maximum potential energy of the well. For this special case Dymski found that the one-dimensional, time-independent Schrödinger equation could be solved in closed form and the solutions normalized. Using these normalized solutions it was concluded that the expectation value of position is equal to the classical time average of position and that the expectation value of the square of the momentum is equal to the classical time average of the square of the momentum.

Although the method of solution is correct, the conclusions regarding the expectation values of position and the square of the momentum are correct *only* in the limit as the quantum number n approaches infinity. If the term resulting from the proper evaluation of the lower limit of the integral in Eq. (16) of Ref. 1 is included, the normalization constant is

$$A_N = \left\{ [X_0 J_{-2/3}(r_n)]^2 - \frac{2^{4/3} X_0^2}{r_n^{4/3} [\Gamma(1/3)]^2} \right\}^{-1/2}, \tag{2}$$

where r_n is the nth root of the one-third-order Bessel function and $\Gamma(\alpha)$ is the gamma function of argument α.

Using Eq. (2) and following Dymski's method of approach, the expectation value of position and the expectation value of the square of the momentum are found to be, respectively,

$$\langle X \rangle = X_0 - \frac{X_0[J_{-2/3}(r_n)]^2}{3\{[J_{-2/3}(r_n)]^2 - R(r_n)\}} \tag{3}$$

and

$$\frac{\langle p^2 \rangle}{2m} = \frac{CX_0[J_{-2/3}(r_n)]^2}{3\{[J_{-2/3}(r_n)]^2 - R(r_n)\}}, \tag{4}$$

where

$$R(r_n) = \frac{2^{4/3}}{r_n^{4/3}[\Gamma(1/3)]^2}.$$

Numerical values for the expectation of position are given in Table I for the first eight quantum states (corresponding to the first eight roots of the one-third-order Bessel function). These values were computed from Eq. (3) for a well of unity width.

As is evident from Eq. (3) and Table I, the expectation value of position depends on the quantum state of the system and approaches the classical value of $(2/3)X_0$ in the limit as r_n (and therefore n) approaches infinity.

TABLE I. Expectation values of position for the first eight quantum states and as n approaches infinity as computed from Eq. (3) with well width $X_0=1$.

n	$\langle X \rangle$	n	$\langle X \rangle$
1	0.459	6	0.579
2	0.522	7	0.585
3	0.548	8	0.589
4	0.562	:	:
5	0.572	∞	0.667[a]

[a] This is also the value of the classical time average of position for $X_0=1$.

Similarly, the expectation value of the square of the momentum is not equal to the classical time average of the square of the momentum, but rather approaches the classical value of $CX_0/3$ as n approaches infinity.

NOBLE M. JOHNSON
AND JOHN N. CHURCHILL

Dep't. of Electrical Engineering
University of California
Davis, California

[1] T. Dymski, Amer. J. Phys. **36**, 54 (1968).

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ANNOUNCEMENTS AND NEWS

Book Reviews

Lectures in Theoretical High Energy Physics. H. H. ALY, Ed. Pp. 439, John Wiley & Sons, Inc., New York, 1968. Price: $17.50 (Reviewed by P. Carruthers.)

The volume under review contains thirteen uncorrelated essays on representative subjects of current interest in particle physics. These articles, originally prepared for presentation in lectures at the American University of Beirut, are of generally high quality. However, the high price and heterogeneous nature of the book will probably discourage most individuals from purchasing this collection. A listing of the authors and titles will perhaps convey the nature of the book: "Selected Topics in Current Algebra" (H. Pietschman); "PC and T Violation" (J. G. Taylor); "Bootstraps and Their Field Quantization" (J. G. Taylor); "The Discrete Symmetries P, C and T" (J. Nilsson); "Ten Years of the Universal V-A Weak Interaction Theory" (E. C. G. Sudarshan); "Compositeness Criteria in Field Theory" (C. R. Hagen); "Topics in Bound State Theory" (D. Lurie); "Some Aspects of the Quark Model for Hadrons" (Riazuddin and A. Q. Sarker); "On the Bosons of Zero Baryon Number as Bound States of Quark–Antiquark Pairs" (J. L. Uretsky); "Inelastic N/D Equations and Regge Theory" (R. L. Warnock); "Some Aspects of High Energy Potential Scattering" (H. H. Aly); "Perturbation Approach for Regular Interactions" (H. J. W. Müller); and "Theory of Symmetry Violation and Gravitation" (J. W. Moffat).

The articles taken together do not comprise an introduction to the subject of elementary particle physics. Many of the essays are sufficiently extensive as to provide interesting reading to graduate students and to nonspecialists in a given area.

[*Peter A. Carruthers is on the physics faculty at Cornell University where he received his Ph.D. in 1961. His primary fields of interest are theory of strong interactions of elementary particles, solid state theory, and transport phenomena at low temperatures.*]