The Oscillator in a Uniform Field

William S. Porter

Citation: American Journal of Physics **33**, 504 (1965); doi: 10.1119/1.1971750
View online: http://dx.doi.org/10.1119/1.1971750
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/33/6?ver=pdfcov
Published by the American Association of Physics Teachers

this fourth equation can be considered, like the others an experimental fact, but it should be remembered that although it was shown above only that four equations are needed, there are still other restrictions and the values of the divergences and curl are not completely independent. There is relativity to consider, and when this is studied it will be found that part of Maxwell's equations are experimental facts (and these will probably not be the parts the students think are the experimental facts) and that part are required by other, more basic, considerations.

The Oscillator in a Uniform Field

WILLIAM S. PORTER
Physics Department, Bucknell University, Lewisburg, Pennsylvania

THE solution of the harmonic oscillator problem in quantum mechanics by the operator method is well known.[1] The technique is not only elegant but it also presents the student with a simple example of the use of the commutation relation between the canonically conjugate position and momentum operators. The insight gained may subsequently be helpful in the study of the properties of angular momentum and, at a more advanced level, in the treatment of the creation and destruction operators for bosons in quantum-field theory.

A second and only slightly less elementary application of the operator method may be made to the problem of a harmonic oscillator subjected to a uniform force along the same direction as the restoring force. Classically, the potential energy function for such a problem is

$$V(x)=\tfrac{1}{2}m\omega^2x^2-Fx, \tag{1}$$

where x = the position coordinate, m = mass of the particle, F = magnitude of the uniform force and ω = the classical angular frequency. The corresponding Hamiltonian is

$$H(p,x)=p^2/2m+\tfrac{1}{2}m\omega^2x^2-Fx, \tag{2}$$

where p is the component of momentum along x. The motion and possible energy of the particle are quantized by interpreting p and x as operators satisfying the commutation rule

$$xp-px=i\hbar, \tag{3}$$

where $\hbar$ = Dirac's constant. The Schrödinger equation

$$H\psi_n=E_n\psi_n \tag{4}$$

is also postulated where E_n is an allowed energy and ψ_n is the wavefunction required to vanish as $x\rightarrow\infty$.

If a new position variable is defined as

$$q=x-F/m\omega^2, \tag{5}$$

then (2) may be rewritten as

$$H(p,q)=\frac{p^2}{2m}+\frac{m\omega^2q^2}{2}-\frac{F^2}{2m\omega^2}, \tag{6}$$

while the commutation rule (3) becomes

$$qp-pq=i\hbar. \tag{7}$$

Except for the last constant term in (6) these last two equations are the same as those occurring in the ordinary oscillator problem. Therefore, the same raising and lowering operators may be defined as in the latter problem and the energy eigenvalues may be obtained by analogous steps which need not be given here.

The allowed energies are found to be

$$E_n=(n+\tfrac{1}{2})\hbar\omega-(F^2/2m\omega)^2, \tag{8}$$

and the presence of the force F is seen to lower each energy level by an amount $F^2/2m\omega^2$. If the oscillating particle carried a charge q and were in a uniform electric field $\mathcal{E}$, then $F=q\mathcal{E}$ and the shift in the energies would be $q^2\mathcal{E}^2/2m\omega^2$. The system would thus have an electric polarizability given by $1/m\omega^2$.

If the explicit representation for p is used

$$p=\frac{\hbar}{i}\frac{d}{dx}=\frac{\hbar}{i}\frac{d}{dq}, \tag{9}$$

the wavefunction for any n may be found as usual. The ground-state function may be obtained most readily and is

$$\psi_0=C\exp(-m\omega q^2/2\hbar), \tag{10}$$

where C = constant. Thus, the wavefunction is Gaussian and is centered around the position $x_0=F/m\omega^2$. The student will appreciate the agreement between this result and his expectations. In a classical model the mass would, under the action of F, assume a new equilibrium position x_0 and oscillate around that point.

Aside from its value in illustrating the operator technique, the present problem is a useful one in which to compare the exact solution for E_n to the result of perturbation theory. If the uniform force F is considered to be a perturbation of the ordinary oscillator whose allowed energies and wavefunctions are known, then, to second order, a standard calculation yields the same result as that given in Eq. (8).[2]

[1] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1947), 3rd ed., p. 136; also Ref. 2, p. 356.
[2] H. Margenau and G. M. Murphy, *The Mathematics of Physics and Chemistry* (D. Van Nostrand, Inc., Princeton, New Jersey, 1943), p. 375.

Poor Man's Optical Bench

DANIEL NAHSHOL AND MOSHE BEN-DOV
Physics Department, Tel-Aviv University, Tel-Aviv, Israel

RECENTLY there arose at the beginners' laboratory of this institute the need for a rather large number of optical benches to be used for a short time only. Money is a consideration in such a case. Some thought and a little experimentation produced a usable optical bench, assembled from parts on store in the laboratory. No money was spent and no workshop time needed. The bench is "student proof" and seems to be as good, if not better, than the usual cheap benches: it does not sag, the slides are secure, and no optical components can fall off. As the parts needed, or similar ones, are probably around every laboratory, the construction may be of somewhat more general interest.

The construction is shown in the accompanying photographs. Three pieces of cylindrical brass bar, A, B, and C, of diameter 12 mm and length about 20 cm are secured