## Molecular Physics

An International Journal at the Interface Between Chemistry and Physics

# An ab initio journey into the electronic structure of $\mathrm{Be}_{3}$ 

## Apostolos Kalemos

To cite this article: Apostolos Kalemos (26 Dec 2023): An ab initio journey into the electronic structure of $\mathrm{Be}_{3}$, Molecular Physics, DOI: 10.1080/00268976.2023.2297822

To link to this article: https://doi.org/10.1080/00268976.2023.2297822


Published online: 26 Dec 2023.


Submit your article to this journal
$\qquad$ Article views: 5


View related articles


View Crossmark data $\sqrt{\top}$

# An ab initio journey into the electronic structure of $\mathrm{Be}_{3}$ 

## Apostolos Kalemos

Department of Chemistry, Laboratory of Physical Chemistry, National and Kapodistrian University of Athens, Athens, Hellas

## ABSTRACT

In the present study, we concisely report on $\mathrm{Be}_{3}$ 's ground and several excited states of singlet, triplet, quintet, and septet spin symmetry at the MRCI/aug-cc-pV5Z computational level. We have constructed potential energy cuts along the bending coordinate under both $\mathrm{C}_{2 v}$ and $\mathrm{C}_{s}$ symmetry constraints. Both linear and bent configurations have been calculated that span an energy range of nearly $160 \mathrm{kcal} / \mathrm{mol}$. The excited states result by promoting one, two or three valence electrons from the ground state's electronic configuration. This electronic promotion destroys its sigma (2center $2 \mathrm{e}^{-}$) bond(s) and the resulting asymmetry is at the very heart of the Jahn-Teller distortion.

## ARTICLE HISTORY

Received 11 November 2023
Accepted 18 December 2023

## KEYWORDS

Berylium clusters; Non adiabatic coupling elements; chemical bond


## 1. Introduction

Despite the 'innocent' rare gas $1 s^{2} 2 s^{2}$ electronic configuration of the ground ( $\left.{ }^{1} \mathrm{~S}\right)$ state Be atom, it is an element of intrinsic complexity [1,2]. Its dimer, $\mathrm{Be}_{2}$, is a covalently
bonded system (see e.g. Refs. [3, 4]) with a very small adiabatic binding energy $\left(\mathrm{D}_{e}(\exp )=934.9 \pm 0.4 \mathrm{~cm}^{-1}, \mathrm{r}_{e}\right.$ $(\exp )=2.445(5) \AA$, Ref. [5]) while its chemical bonding puzzled the scientific community for nearly 60 years.

Despite the above peculiar characteristics, beryllium is a highly toxic metal with a melting point at about $1300{ }^{\circ} \mathrm{C}$ [6].

It seems that its first theoretical description is a 1975 study by Bulski who studied the three-body contributions to the exchange repulsion [7]. At the ab initio level its first post-HF study appears to be the MRCI/[7s4p2d] calculations by Harrison and Handy [8] who in 1986 predicted a binding energy of $19(21) \mathrm{kcal} / \mathrm{mol}$ at the $\mathrm{CI}(+\mathrm{Q})$ level. Some years later, Lee, Rendell, and Taylor [9] calculated Be and Mg clusters with a variety of computational methods. Their best results for $\mathrm{Be}_{3}$ are $\mathrm{r}_{e}=4.199$ bohr and $\mathrm{D}_{e}=22.4 \mathrm{kcal} / \mathrm{mol}$ at the $\mathrm{MRCI}(0.025) /[5321]$ level while their best estimate for the binding energy is $24 \mathrm{kcal} / \mathrm{mol}$. In 2000, Kaplan, Roszak, and Leszczynski [10] studied the nature of the bonding in $\mathrm{Be}_{3}, \mathrm{Mg}_{3}$, and $\mathrm{Ca}_{3}$ and concluded that it is due to two body localised dispersion forces and three body delocalised exchange forces. In a $\mathrm{FCI} /[3 s 2 p 1 d]$ study by Junquera-Hernández et al. [11] a $\mathrm{D}_{e}=17.2 \mathrm{kcal} / \mathrm{mol}$ is reported for its most stable $\mathrm{D}_{3 h}$ configuration. The same authors reported a similar $\mathrm{FCI} /[3 s 2 p 1 d]$ study for several vertical singlet excited states [12]. An AQCC calculation based on a large CASSCF reference wavefunction gave a $\mathrm{D}_{e}=27.3$ $\mathrm{kcal} / \mathrm{mol}\left(\right.$ aug-cc-pV5Z) at $\mathrm{r}_{e}=4.101$ bohr (cc-pV5Z) [13]. The authors also performed an EPFL analysis that showed the crucial role of the $2 p$ orbitals in the formation of the chemical bond. By a similar computational strategy (CASSCF + AQCC) Ramírez-Solís and Novaro [14] attributed the large binding energy of the trimer (26.1 $\mathrm{kcal} / \mathrm{mol}$ ) to non-additive collective effects that amount to nearly $94 \%$ of the $\mathrm{D}_{e}$.

The experimental and theoretical corpus on Be clusters has been recently reviewed in [15] and [16], while the latest experimental work [17] on $\mathrm{Be}_{3}$ and $\mathrm{Be}_{4}$ proves the enormous academic interest on its chemistry.

We have gained considerable experience on Be bonding through our recent studies on different species such as $\mathrm{Be}_{2}$ [3], beryllium oxides [18], $\mathrm{BeF}^{-}$[19], $\mathrm{Be}_{3}{ }^{-}$[20], and finally $\mathrm{BeH}^{-}$[4]. A common trait in all of the above species is the participation of the excited $2 s^{1} 2 p^{1}$ configuration $\left(\Delta \mathrm{E}\left({ }^{3} \mathrm{P} \leftarrow{ }^{1} \mathrm{~S}\right)=2.725 \mathrm{eV}\right.$ and $\Delta \mathrm{E}\left({ }^{1} \mathrm{P} \leftarrow{ }^{1} \mathrm{~S}\right)=$ 5.277 eV , Ref. [21]) in the formation of both covalent and dative bonds; see e.g. Ref. [4] for an adiabatic to diabatic transformation that explicitly shows the presence of the excited $2 s^{1} 2 p^{1}$ configuration in the bonding process.

We have studied $\mathrm{Be}_{3}$ 's ground state in Ref. [3] and its anion, $\mathrm{Be}_{3}{ }^{-}$, in Ref. [20]. In the latter study, eleven electronic states have been detailed that were associated to five states of its parental neutral species. Its ground state is of equilateral geometry $\left(\mathrm{D}_{3 h}\right)$ and of $\tilde{X}^{1} \mathrm{~A}^{\prime}{ }_{1}$ symmetry. Its bonding results when three excited ${ }^{3} \mathrm{P} \mathrm{Be}$ atoms form sigma (type) bonds with each other. The ground anionic
state results when an additional electron is drafted to the $\pi$ system $\left({ }^{2} \mathrm{~B}_{1}\right)$ of the neutral species (EA $=1.379 \mathrm{eV}$, [20]) while its first excited state results when the minus charge resides in the molecular plane $\left({ }^{2} \mathrm{~A}_{1}, \Delta \mathrm{E}\left({ }^{2} \mathrm{~A}_{1} \leftarrow\right.\right.$ $\left.{ }^{2} \mathrm{~B}_{1}\right)=0.359 \mathrm{eV}$ ). In our present study we focus exclusively on the neutral $\mathrm{Be}_{3}$ system with the aim to delve more deeply into its electronic structure by elucidating as many states as possible.

## 2. Computational details

Beryllium trimer is a six-electron valence system that can form states of singlet, triplet, quintet, and septet spin multiplicity. We have constructed potential energy profiles along the bending coordinate for the low-lying states of each space-spin symmetry by optimising a $\mathrm{C}_{2 v}$ geometry under both $\mathrm{C}_{2 v}$ and $\mathrm{C}_{s}$ symmetry constraints. Our zeroth-order wavefunction is of the complete active space self-consistent field (CASSCF) type whose active space is composed of 12 valence orbitals converging at infinity to the atomic $2 s$ and $2 p$ Be orbitals. Additional dynamic correlation was extracted through the internally contracted multi-reference configuration interaction (MRCI) method built upon the CASSCF reference. The aug - cc - pV5Z basis set [22] was employed and the MOLPRO computational package [23] was used.

## 3. Results and discussion

A panoramic view of the calculated states is displayed in Figure 1 while numerical results are gathered in Table 1. In Figure 1 we show potential energy cuts along the bending coordinate when a $\mathrm{C}_{2 v}$ geometry is optimised under $\mathrm{C}_{2 v}$ symmetry constraints. Different spin multiplicities are displayed in different colour; black for singlets, red for triplets, green for quintets, and blue for septets while solid lines are used for energy profiles that transform as $\mathrm{A}^{\prime}$ under $\mathrm{C}_{s}$ symmetry constraints and dashed lines for those states that transform as $\mathrm{A}^{\prime \prime}$. Only the lowest energy states of each space-spin symmetry label have been calculated at the MRCI computational level. A vertical line at 60 degrees highlights the symmetry imposed conical intersections of Jahn-Teller (JT) character. For selected (space-spin) states we have also calculated the energy cuts at $\mathrm{C}_{s}$ symmetry constraints (see Figure 2). The latter $\mathrm{C}_{s}$ profiles are more appropriate for the study of nuclear dynamics although the crossing ones $\left(\mathrm{C}_{2 v}\right)$ turn out to be more easily manipulated when no adiabatic effects are in order.

The $\mathrm{Be}_{3}$ ground state is of $\mathrm{D}_{3 h}$ geometry and ${ }^{1} \mathrm{~A}_{1}$ $\left(\mathrm{C}_{2 v}\right)$ symmetry and it results from the interaction of three excited Be atoms that form 2 c (enter) $-2 \mathrm{e}^{-}$ bonds with their adjacent atoms (see Refs. [3] and [4]).


Figure 1. $\mathrm{MRCl} / \mathrm{aug}-\mathrm{cc}-\mathrm{pV} 5 \mathrm{Z}$ potential energy profiles of $\mathrm{Be}_{3}$ states of singlet (black), triplet (red), quintet (green), and septet (blue) spin symmetry along the bending coordinate under $\mathrm{C}_{2 v}$ symmetry constraints. Solid lines are used for energy profiles that transform as $A^{\prime}$ under $C_{S}$ symmetry constraints and dashed lines for those states that transform as $A^{\prime \prime}$.

The excited nature of the atomic constituents is also revealed by the calculation of the non adiabatic coupling matrix elements (NACME) along the asymmetric dissociation path $\mathrm{Be}_{3} \rightarrow \mathrm{Be}_{2}+\mathrm{Be}$ (see Figure 3). Its ground state dissociates adiabatically to ground state fragments, i.e. $\mathrm{Be}_{2}\left(X^{1} \Sigma_{g}^{+}\right)+\mathrm{Be}\left({ }^{1} \mathrm{~S}\right)$. But around its equilibrium configuration, there are intense NACMEs with the 2nd and $3 \mathrm{rd}^{1} \mathrm{~A}^{\prime}$ states that converge asymptotically to 3 ${ }^{1} \Pi_{g}+{ }^{1} \mathrm{~S}$ and $6{ }^{1} \Pi_{u}+{ }^{1} \mathrm{~S}$, respectively. The latter $\mathrm{Be}_{2}$ molecular states result from the combination of Be $\left({ }^{1} \mathrm{~S}\right)+\mathrm{Be}\left({ }^{1} \mathrm{P}\right)[3]$. So due to the centrosymmetric character of the diatomic and the $\mathrm{C}_{3}$ symmetry of the triatomic the excited $2 s^{1} 2 p^{1}$ nature of the atomic centers in $\mathrm{Be}_{3}$ occurs naturally. The ground state is bound by $24.5(25.5)$ $\mathrm{kcal} / \mathrm{mol}$ at the $\mathrm{MRCI}(+\mathrm{Q})$ computational level with respect to three $\mathrm{Be}\left({ }^{1} \mathrm{~S}\right)$ atoms at $\mathrm{r}_{e}=2.203 \AA$. Our numerical data compare well with the previous reported values of $24\left(\mathrm{r}_{e}=2.222 \AA\right)$ [9], $27.3\left(\mathrm{r}_{e}=2.170 \AA\right.$ ) [13], and $26.1 \mathrm{kcal} / \mathrm{mol}\left(\mathrm{r}_{e}=2.170 \AA\right)$ [ 14$]$.

A vertical excitation from the ground state ends up to the conical intersection of two curves of ${ }^{3} \mathrm{~B}_{1}$ and ${ }^{3} \mathrm{~A}_{2}$ symmetry (see Figure 1) that under $\mathrm{C}_{s}$ transform into
a double minimum ${ }^{3} \mathrm{~A}^{\prime \prime}$ curve and another ${ }^{3} \mathrm{~A}^{\prime \prime}$ curve whose minimum sits on the energy maximum of its JT companion (see Figure 2). On the lowest ${ }^{3} \mathrm{~A}^{\prime \prime}$ curve the minimum at $68.7^{\circ}$ (see Table 1 ) is of ${ }^{3} \mathrm{~B}_{1}$ symmetry and correlates to a ${ }^{3} \Pi_{u}$ linear configuration while the minimum at $53.9^{\circ}$ is of ${ }^{3} \mathrm{~A}_{2}$ symmetry and correlates to a linear ${ }^{3} \Pi_{g}$ state. It is interesting to observe the equilibrium configurations of these two triplet states and compare them to those of its ground state:

$$
\begin{aligned}
\left|\tilde{X}^{1} \mathrm{~A}_{1}\right\rangle \cong & 0.87\left|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{2}^{2}\right\rangle, \\
\left|4^{3} \mathrm{~B}_{1}\right\rangle \cong & 0.90\left|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{2}\right\rangle-0.20\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{2} 1 b_{1}^{1}\right\rangle \\
& -0.11\left|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1} 2 b_{2}^{2}\right\rangle, \quad \text { and } \\
\left|5^{3} \mathrm{~A}_{2}\right\rangle \cong & 0.89\left|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle-0.22\left|1 a_{1}^{2} 3 a_{1}^{2} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle .
\end{aligned}
$$

The triplet states result from the excitation of a $2 a_{1}$ $\left(\tilde{X}^{1} \mathrm{~A}_{1} \rightarrow 4{ }^{3} \mathrm{~B}_{1}\right)$ or a $1 b_{2}\left(\tilde{X}^{1} \mathrm{~A}_{1} \rightarrow 5^{3} \mathrm{~A}_{2}\right)$ electron to an orbital of $b_{1}$ symmetry (perpendicular to the molecular plane), respectively. The other contributions offer a GVBtype correlation to the $1 b_{2}\left(4^{3} \mathrm{~B}_{1}\right)$ or $2 a_{1}\left(5^{3} \mathrm{~A}_{2}\right)$ electron pairs. In chemical terms one of the $2 \mathrm{c}-2 \mathrm{e}^{-}$sigma bonds of the ground state becomes a $2 \mathrm{c}-1 \mathrm{e}^{-}$sigma bond and this creates a bonding asymmetry which is the chemical origin of the JT distortion.

The second excited state that can be reached vertically is a manifold of two JT states of (7) ${ }^{3} \mathrm{~A}_{1}$ and (8) ${ }^{3} \mathrm{~B}_{2}$ symmetry. Under $\mathrm{C}_{s}$ symmetry constraints these two crossing curves give rise to two ${ }^{3} \mathrm{~A}^{\prime}$ profiles that correlate to ${ }^{3} \Sigma_{u}^{+}$and ${ }^{3} \Pi_{u}$ linear states, respectively. Once again the lowest ${ }^{3} \mathrm{~A}^{\prime}$ sheet displays two minima of ${ }^{3} \mathrm{~A}_{1}$ (at $73.9^{\circ}$ ) and ${ }^{3} \mathrm{~B}_{2}$ (at $54.3^{\circ}$ ) symmetry, respectively. This pair of ${ }^{3} \mathrm{~A}^{\prime}$ states display not only a symmetry imposed conical intersection but also an accidental one at around $112^{\circ}$; see Figures 1 and 2. Once again the equilibrium configurations provide an insight into their bonding situation. These are

$$
\begin{aligned}
\left|7^{3} \mathrm{~A}_{1}\right\rangle \cong & 0.88\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{2}^{2}\right\rangle+0.20\left|1 a_{1}^{2} 2 a a_{1}^{2} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle \\
& -0.16\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 2 b_{2}^{2}\right\rangle-0.15\left|1 a_{1}^{2} 3 a_{1}^{2} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle,
\end{aligned}
$$

and

$$
\left|8^{3} \mathrm{~B}_{2}\right\rangle \cong 0.89\left|1 a_{1}^{2} 2 a_{1}^{2} 3 a_{1}^{1} 1 b_{2}^{1}\right\rangle-0.20\left|1 a_{1}^{2} 3 a_{1}^{1} 4 a_{1}^{2} 1 b_{2}^{1}\right\rangle .
$$

It is evident that the $7{ }^{3} \mathrm{~A}_{1}$ state results mainly from the closed shell ground state configuration $\left(\left|\tilde{X}^{1} \mathrm{~A}_{1}\right\rangle \cong\right.$ $\left.0.87\left|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{2}^{2}\right\rangle\right)$ by exciting a $2 a_{1}$ electron to the $3 a_{1}$ orbital while the $8{ }^{3} \mathrm{~B}_{2}$ state results when a $1 b_{2}$ electron gets also promoted to the $3 a_{1}$ orbital.

The next state to be considered is of ${ }^{5} \mathrm{~A}_{2}\left(\left|12^{5} \mathrm{~A}_{2}\right\rangle \cong\right.$ $\left.0.94\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle\right)$ symmetry that relates to a ${ }^{5} \Sigma_{u}^{-}$ linear configuration. It results from the ground state configuration by promoting an electron from $2 a_{1}$ to $3 a_{1}$ and

Table 1. Energies E (hartree), bond distances $\mathrm{r}_{\mathrm{e}}(\AA)$, bond angles $\theta_{\mathrm{Be}_{2} \mathrm{Be}_{1} \mathrm{Be}_{3}}$ (degrees), energy gaps $\Delta \mathrm{E}(\mathrm{kcal} / \mathrm{mol})$ and main configurations of the $\mathrm{Be}_{3}$ states presently studied at the $\mathrm{MRCl} / a u g-\mathrm{cc}-\mathrm{pV} 5 \mathrm{Z}$ computational level. $\mathrm{AC}_{2 \mathrm{v}}$ notation is used for the bent configurations, $r_{\mathrm{e}}$ refers to the bond length between the middle $\left(\mathrm{Be}_{1}\right)$ and the terminal $\left(\mathrm{Be}_{2,3}\right)$ atoms while linear configurations lie on the $y$ axis with $\mathrm{Be}_{1}$ placed on the origin. At the $\mathrm{MRCI} / \mathrm{aug}-\mathrm{cc}-\mathrm{pV} 5 \mathrm{Z}$ level the atomic Be energies are $\mathrm{E}\left({ }^{1} \mathrm{~S}\right)=-14.619196 \mathrm{E}_{\mathrm{h}}$ and $\mathrm{E}\left({ }^{3} \mathrm{P}\right)=-14.518$ $666 \mathrm{E}_{\mathrm{h}}$.

| State | -E | $\mathrm{r}_{e}$ | $\theta_{\mathrm{Be}_{2} \mathrm{Be}_{1} \mathrm{Be}_{3}}$ | $\Delta \mathrm{E}$ | Main configurations |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\tilde{\chi}^{1} \mathrm{~A}_{1}$ | 43.898160 | 2.203 | 60.0 | 0.0 | $\left\|\tilde{X}^{1} \mathrm{~A}_{1}\right\rangle \cong 0.87\left\|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{2}^{2}\right\rangle$ |
| $2^{1} \Sigma_{g}^{+}$ | 43.882018 | 2.207 | 180.0 | 10.1 | $\left\|2^{1} \Sigma_{g}^{+}\right\rangle \cong\left\|1 a_{1}^{2}\left(0.77 \times 2 a_{1}^{2}-0.53 \times 2 b_{2}^{2}\right) 1 b_{2}^{2}\right\rangle$ |
| $3^{3} \Sigma_{u}^{+}$ | 43.874237 | 2.145 | 180.0 | 15.0 | $\left\|3^{3} \Sigma_{u}^{+}\right\rangle \cong 0.95\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{2}^{2} 2 b_{2}^{1}\right\rangle$ |
| $4^{3} \mathrm{~B}_{1}$ | 43.872020 | 1.991 | 68.7 | 16.4 | $\left\|4^{3} \mathrm{~B}_{1}\right\rangle \cong\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1}\left(0.90 \times 1 b_{2}^{2}-0.20 \times 3 a_{1}^{2}\right)\right\rangle$ |
| $5^{3} \mathrm{~A}_{2}$ | 43.871463 | 2.134 | 53.9 | 16.8 | $\left\|5^{3} \mathrm{~A}_{2}\right\rangle \cong\left\|1 a_{1}^{2}\left(0.89 \times 2 a_{1}^{2}-0.22 \times 3 a_{1}^{2}\right) 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $6^{3} \Pi_{u}$ | 43.862156 | 2.071 | 180.0 | 22.6 | $\left\|6^{3} \Pi_{u}\left({ }^{3} \mathrm{~B}_{1}\right)\right\rangle \cong 0.88\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{2}\right\rangle-0.22\left\|1 a_{1}^{2} 1 b_{2}^{2} 2 b_{2}^{1} 1 a_{2}^{1}\right\rangle$ |
| $7^{3} \mathrm{~A}_{1}$ | 43.858070 | 2.079 | 73.9 | 25.2 | $\left\|7^{3} \mathrm{~A}_{1}\right\rangle \cong 0.88\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{2}^{2}\right\rangle$ |
| $8^{3} \mathrm{~B}_{2}$ | 43.856396 | 2.239 | 54.3 | 26.2 | $\left\|8^{3} \mathrm{~B}_{2}\right\rangle \cong 0.89\left\|1 a_{1}^{2} 2 a_{1}^{2} 3 a_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $9^{1} \Pi_{u}$ | 43.853637 | 2.054 | 180.0 | 27.9 | $\left\|9^{1} \Pi_{u}\left({ }^{1} \mathrm{~B}_{1}\right)\right\rangle \cong 0.85\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 \bar{b}_{1}^{1} 1 b_{2}^{2}\right\rangle-0.31\left\|1 a_{1}^{2} 1 b_{2}^{2} 2 b_{2}^{1} 1 \bar{a}_{2}^{1}\right\rangle$ |
| $10^{1} \mathrm{~B}_{1}$ | 43.851745 | 1.987 | 71.5 | 29.1 | $\left\|10^{1} \mathrm{~B}_{1}\right\rangle \cong 0.89\left\|1 a_{1}^{2} 2 a a_{1}^{1} 1 \bar{b}_{1}^{1} 1 b_{2}^{2}\right\rangle-0.18\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{2} 1 \bar{b}_{1}^{1}\right\rangle$ |
| $11^{1} \mathrm{~A}_{2}$ | 43.850494 | 2.155 | 53.1 | 29.9 | $\left\|11^{1} \mathrm{~A}_{2}\right\rangle \cong 0.89\left\|1 a_{1}^{2}\left(0.89 \times 2 a_{1}^{2}-0.21 \times 3 a_{1}^{2}\right) 1 b_{1}^{1} 1 \bar{b}_{2}^{1}\right\rangle$ |
| $12^{5} \mathrm{~A}_{2}$ | 43.848122 | 1.998 | 60.0 | 31.4 | $\left\|12^{5} \mathrm{~A}_{2}\right\rangle \cong 0.94\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $13^{1} \mathrm{~A}_{1}$ | 43.837108 | 2.026 | 80.8 | 38.3 | $\left\|13^{1} \mathrm{~A}_{1}\right\rangle \cong 0.78\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 \bar{a}_{1}^{1} 1 b_{2}^{2}\right\rangle-0.21\left\|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{2}^{2}\right\rangle-0.21\left\|1 a_{1}^{2} 3 a_{1}^{2} 1 b_{2}^{2}\right\rangle$ |
| $14^{1} \Pi_{g}$ | 43.836635 | 2.050 | 180.0 | 38.6 | $\left\|14^{1} \Pi_{g}\left({ }^{1} \mathrm{~B}_{2}\right)\right\rangle \cong 0.74\left\|1 a_{1}^{2} 3 a_{1}^{1} 1 b_{2}^{2} 2 \bar{b}_{2}^{1}\right\rangle-0.54\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{2}^{2} 3 \bar{b}_{2}^{1}\right\rangle$ |
| $15^{3} \Pi_{g}$ | 43.836303 | 2.047 | 180.0 | 38.8 | $\left\|15^{3} \Pi_{g}\left({ }^{3} \mathrm{~A}_{2}\right)\right\rangle \cong 0.74\left\|1 a_{1}^{2} 1 b_{1}^{1} 1 b_{2}^{2} 2 b_{2}^{1}\right\rangle+0.54\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{2}^{2} 1 a_{2}^{1}\right\rangle$ |
| $16^{3} \mathrm{~B}_{1}$ | 43.835733 | 1.932 | 180.0 | 39.2 | $\left\|16^{3} \mathrm{~B}_{1}\right\rangle \cong 0.88\left\|1 a_{1}^{2} 3 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{2}\right\rangle$ |
| ${ }_{17}{ }^{1} \mathrm{~B}_{2}$ | 43.835717 | 2.338 | 49.1 | 39.2 | $\left\|17^{1} \mathrm{~B}_{2}\right\rangle \cong 0.91\left\|1 a_{1}^{2} 2 a_{1}^{2} 3 a_{1}^{1} 1 \bar{b}_{2}^{1}\right\rangle$ |
| $18^{3} \mathrm{~B}_{2}$, (global) | 43.832128 | 2.609 | 45.0 | 41.4 | $\left\|18^{3} \mathrm{~B}_{2}\right\rangle \cong 0.66\left\|1 a_{1}^{2} 2 a_{1}^{2} 5 a_{1}^{1} 1 b_{2}^{1}\right\rangle-0.57\left\|1 a_{1}^{2} 2 a_{1}^{2} 3 a_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $18^{3} \mathrm{~B}_{2, ~(\text { local }}$ | 43.825449 | 2.126 | 70.0 | 45.6 | $\left\|18^{3} \mathrm{~B}_{2}\right\rangle \cong 0.63\left\|1 a_{1}^{2} 2 a a_{1}^{1} 1 b_{2}^{2} 2 b_{2}^{1}\right\rangle-0.50\left\|1 a_{1}^{2} 2 a_{1}^{2} 3 a_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $193^{3} \mathrm{~A}_{1}$ | 43.814754 | 2.256 | 60.0 | 52.3 | $\left\|19^{3} \mathrm{~A}_{1}\right\rangle \cong 0.69\left\|1 a_{1}^{2} 2 a_{1}^{1} 4 a_{1}^{1} 1 b_{2}^{2}\right\rangle-0.50\left\|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| $20^{3} \mathrm{~A}_{1}$ | 43.811885 | 2.213 | 63.0 | 54.1 | $\left\|20^{3} \mathrm{~A}_{1}\right\rangle \cong 0.75\left\|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| ${ }^{21}{ }^{5} \mathrm{~B}_{1}$ | 43.811544 | 2.026 | 70.0 | 54.4 | $\left\|21^{5} \mathrm{~B}_{1}\right\rangle \cong 0.94\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| $22^{5} \mathrm{~A}_{2}$ | 43.809653 | 2.153 | 55.5 | 55.5 | $\left\|22^{5} \mathrm{~A}_{2}\right\rangle \cong 0.93\left\|1 a_{1}^{2} 2 a_{1}^{1} 4 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $23^{1} \mathrm{~B}_{2}$ | 43.808758 | 1.963 | 60.0 | 56.1 | $\left\|23^{1} \mathrm{~B}_{2}\right\rangle \cong 0.89\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{2} 1 \bar{b}_{2}^{1}\right\rangle$ |
| $24^{5} \mathrm{~A}_{1}$ | 43.806526 | 2.090 | 72.6 | 57.5 | $\left\|24^{5} \mathrm{~A}_{1}\right\rangle \cong 0.96\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| ${ }^{25}{ }^{5} \mathrm{~B}_{2}$ | 43.804160 | 2.266 | 53.9 | 59.0 | $\left\|25^{5} \mathrm{~B}_{2}\right\rangle \cong 0.96\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 4 a_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $265^{1} \mathrm{~A}_{2}$ | 43.800618 | 2.058 | 58.0 | 61.2 | $\left\|26^{1} \mathrm{~A}_{2}\right\rangle \cong 0.81\left\|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 \bar{b}_{1}^{1} 1 \bar{b}_{2}^{1}\right\rangle$ |
| $27^{3} \mathrm{~B}_{1}$ | 43.787389 | 2.076 | 60.0 | 69.5 |  |
| ${ }^{28}{ }^{5} \Pi_{u}$ | 43.787296 | 2.097 | 180.0 | 69.6 | $\left\|28^{5} \Pi_{u}\left({ }^{5} \mathrm{~B}_{1}\right)\right\rangle \cong 0.92\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| $29^{3} \mathrm{~A}_{1}$ | 43.779330 | 2.147 | 180.0 | 74.6 |  |
| $30^{1} \Pi_{g}$ | 43.758164 | 2.202 | 180.0 | 87.8 |  |
| $31^{5} \Pi_{g}$ | 43.753845 | 2.192 | 180.0 | 90.6 | $\left\|31^{5} \Pi_{g}\left({ }^{5} \mathrm{~A}_{2}\right)\right\rangle \cong 0.70\left\|1 a_{1}^{1} 2 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{2} 2 b_{2}^{1}\right\rangle+0.64\left\|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1} 1 a_{2}^{1}\right\rangle$ |
| $32^{7} \mathrm{~B}_{1}$ | 43.690419 | 2.071 | 95.0 | 130.4 | $\left\|32^{7} \mathrm{~B}_{1}\right\rangle \cong 0.98\left\|1 a_{1}^{1} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| $3^{3}{ }^{7} \mathrm{~A}_{2}$ | 43.683649 | 2.376 | 50.0 | 134.6 | $\left\|33^{7} \mathrm{~A}_{2}\right\rangle \cong 0.98\left\|1 a_{1}^{1} 2 a_{1}^{1} 3 a_{1}^{1} 4 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle$ |
| $34^{7} \mathrm{~A}_{1}$ | 43.637721 | 2.564 | 60.0 | 163.4 | $\left\|34^{7} \mathrm{~A}_{1}\right\rangle \cong 0.99\left\|1 a_{1}^{1} 2 a_{1}^{1} 3 a_{1}^{1} 5 a_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |
| $35^{7} \mathrm{~B}_{1}$ | 43.633629 | 2.419 | 60.0 | 166.0 | $\left\|35^{7} \mathrm{~B}_{1}\right\rangle \cong 0.88\left\|1 a_{1}^{1} 2 a_{1}^{1} 5 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle+0.34\left\|1 a_{1}^{1} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle$ |

another one from $1 b_{2}$ to $1 b_{1}$ while retaining the singly electrons highly coupled.

Slightly above there is a JT conical intersection that gives rise to two branches of ${ }^{1} \mathrm{~B}_{1} \quad\left(\left|10^{1} \mathrm{~B}_{1}\right\rangle \cong\right.$ $\left.0.89\left|1 a_{1}^{2} 2 a_{1}^{1} 1 \bar{b}_{1}^{1} 1 b_{2}^{2}\right\rangle-0.18\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{2} 1 \bar{b}_{1}^{1}\right\rangle\right)$ and ${ }^{1} \mathrm{~A}_{2}$ $\left(\left|11^{1} \mathrm{~A}_{2}\right\rangle \cong 0.89\left|1 a_{1}^{2} 2 a_{1}^{2} 1 b_{1}^{1} 1 \bar{b}_{2}^{1}\right\rangle-0.20\left|1 a_{1}^{2} 3 a_{1}^{2} 1 b_{1}^{1} 1 \bar{b}_{2}^{1}\right\rangle\right)$ symmetry that relate to a ${ }^{1} \Pi_{u}$ and ${ }^{1} \Pi_{g}$ linear states, respectively, while they transform as ${ }^{1} \mathrm{~A}^{\prime \prime}$ under $\mathrm{C}_{s}$ symmetry constraints (see Figure 2). By inspecting their equilibrium configurations we can see that these states are the
singlet analogs of the $4{ }^{3} \mathrm{~B}_{1}$ and $5^{3} \mathrm{~A}_{2}$ states, respectively (see above).

Another pair of JT-related states are the couple of 13 ${ }^{1} \mathrm{~A}_{1}$ (with a minimum at $80.8^{\circ}$ that relates to a ${ }^{1} \Pi_{u}$ linear state) and $17{ }^{1} \mathrm{~B}_{2}$ (with a minimum at $49.1^{\circ}$ that relates to a ${ }^{1} \Pi_{g}$ linear state) states that are the Renner - Teller companions of the above discussed $10{ }^{1} \mathrm{~B}_{1}$ and $11{ }^{1} \mathrm{~A}_{2}$ states, respectively.

Close enough there are two pairs of quintet spin symmetry states connected through a JT conical intersection.


Figure 2. $\mathrm{MRCl} / \mathrm{aug}-\mathrm{cc}-\mathrm{pV} 5 \mathrm{Z}$ potential energy profiles of $\mathrm{Be}_{3}$ states of singlet (black), triplet (red), and quintet (green) spin symmetry along the bending coordinate under $\mathrm{C}_{s}$ symmetry constraints.

The (energy) lowest pair splits off to a $21^{5} \mathrm{~B}_{1}$ $\left(\left|21^{5} \mathrm{~B}_{1}\right\rangle \cong 0.94\left|1 a_{1}^{2} 2 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle\right)$ and a $22{ }^{5} \mathrm{~A}_{2}\left(\mid 22^{5}\right.$ $\left.\left.\mathrm{A}_{2}\right\rangle \cong 0.93\left|1 a_{1}^{2} 2 a_{1}^{1} 4 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle\right)$ states. The other JT pair splits to $24{ }^{5} \mathrm{~A}_{1}\left(\left|24^{5} \mathrm{~A}_{1}\right\rangle \cong 0.96\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle\right)$ and $25^{5} \mathrm{~B}_{2}\left(\left|25^{5} \mathrm{~B}_{2}\right\rangle \cong 0.96\left|1 a_{1}^{2} 2 a_{1}^{1} 3 a_{1}^{1} 4 a_{1}^{1} 1 b_{2}^{1}\right\rangle\right)$ states. These four quintet states are interconnected at the Renner - Teller limit, i.e. ${ }^{5} \Pi_{u}\left({ }^{5} \mathrm{~A}_{1},{ }^{5} \mathrm{~B}_{1}\right)$ and ${ }^{5} \Pi_{g}\left({ }^{5} \mathrm{~A}_{2}\right.$, ${ }^{5} \mathrm{~B}_{2}$ ).

The potential curves presented in both Figures 1 and 2 represent only a small fraction of all possible states of singlet, triplet, and quintet spin symmetry. We have only calculated the ground and the lowest two excited states of each space-spin symmetry. States of septet spin symmetry are also possible but they represent a chemical curiosity since all six valence electrons should be highly coupled. So, it is a surprise to us the existence of such a rich manifold of bound septet states. The energy lowest septet state has a very shallow minimum and it is of (32) ${ }^{7} \mathrm{~B}_{1}\left(\left|32^{7} \mathrm{~B}_{1}\right\rangle \cong 0.98\left|1 a_{1}^{1} 2 a_{1}^{1} 3 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1} 2 b_{2}^{1}\right\rangle\right),{ }^{7} \Sigma_{g}^{-}$at linearity symmetry. The deepest curve belongs to the 33 ${ }^{7} \mathrm{~A}_{2}\left(\left|33^{7} \mathrm{~A}_{2}\right\rangle \cong 0.98\left|1 a_{1}^{1} 2 a_{1}^{1} 3 a_{1}^{1} 4 a_{1}^{1} 1 b_{1}^{1} 1 b_{2}^{1}\right\rangle\right)$ state that is the JT companion of the $32{ }^{7} \mathrm{~B}_{1}$ one. In addition to the symmetry-imposed intersection at 60 degrees, the ${ }^{7} \mathrm{~A}_{2}$


Figure 3. NACMEs at the SACASSCF/aug - $\mathrm{cc}-\mathrm{pV} 5 \mathrm{Z}$ computational level along the asymmetric ( $A^{\prime}$ ) dissociation path $B e_{3}\left(D_{3 h}\right)$ $\rightarrow \mathrm{Be}_{2}+\mathrm{Be}$ while retaining the $\mathrm{Be}_{2}$ distance equal to the value of the ground state trimer. All states are of ${ }^{1} \mathrm{~A}_{1}$ symmetry at their $\mathrm{D}_{3 h}$ configuration.
state suffers another intersection at around 75 degrees with a second ${ }^{7} \mathrm{~A}_{2}$ state. With respect to the closed shell ground state a septet state results by fully uncoupling all its valence electrons.

## 4. Conclusions

We have studied at the highest possible level (MRCI/ aug-cc - pV5Z) the $\mathrm{Be}_{3}$ species. We report data on 35 states of singlet, triplet, quintet, and finally septet spin symmetry at both linear and bent configurations spanning an energy range of approximately $160 \mathrm{kcal} / \mathrm{mol}$. Its ground state is of closed shell character and results from the interaction of three excited $\left(2 s^{1} 2 p^{1}\right) \mathrm{Be}$ atoms. It is bound by circa $25 \mathrm{kcal} / \mathrm{mol}$ with respect to three Be $\left({ }^{1} \mathrm{~S}\right)$ atoms. All other states result by exciting one, two, and/or three valence electrons to higher orbitals that either belong to the molecular plane or are perpendicular to it. This electron excitation breaks the $2 \mathrm{c}-2 \mathrm{e}^{-}$ bond(s) of the ground state and the resulting asymmetry
is at the very heart of the JT distortion. A characteristic feature of the present work is the existence of septet spin states that result by fully uncoupling all six valence electrons. Their bound nature is due to the stabilisation of the kinetic energy within the molecular space provided.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

## References

[1] R.E. Watson, Phys. Rev. 119, 170 (1960). doi:10.1103/ PhysRev.119.170
[2] A.W. Weiss, Phys. Rev. 122, 1826 (1961). doi:10.1103/ PhysRev.122.1826
[3] A. Kalemos, J. Chem. Phys. 145, 214302 (2016). doi:10.1063/1.4967819
[4] A. Kalemos. Mol. Phys. 121, e2230315 (2023). doi:10.1080/00268976.2023.2230315
[5] V.V. Meshkov, A.V. Stolyarov, M.C. Heaven, C. Haugen, and R.J. LeRoy, J. Chem. Phys. 140, 064315 (2014). doi:10.1063/1.4864355
[6] N.N. Greenwood and A. Earnshaw, Chemistry of the Elements (Butterworth-Heinemann, Oxford, 1997).
[7] M. Bulski, Mol. Phys. 29, 1171 (1975). doi:10.1080/00268 977500100991
[8] R.J. Harrison and N.C. Handy, Chem. Phys. Lett. 123, 321 (1986). doi:10.1016/0009-2614(86)80080-X
[9] T.J. Lee, A.P. Rendell and P.R. Taylor, J. Chem. Phys. 92, 489 (1990). doi:10.1063/1.458570
[10] I.G. Kaplan, S. Roszak and J. Leszczynski, J. Chem. Phys. 113, 6245 (2000). doi:10.1063/1.1287835
[11] J.M. Junquera-Hernández, J. Sánchez-Marín, G.L. Bendazzoli, and S. Evangelisti, J. Chem. Phys. 120, 8405 (2014). doi:10.1063/1.1695328
[12] J.M. Junquera-Hernández, J. Sánchez-Marín, G.L. Bendazzoli, and S. Evangelisti, J. Chem. Phys. 121, 7103 (2014). doi:10.1063/1.1792152
[13] J.I. Amaro-Estrada, A. Scemama, M. Caffarel, and A. Ramírez-Solís, J. Chem. Phys. 135, 104311 (2011). doi:10.1063/1.3635403
[14] A. Ramírez-Solís and O. Novaro, Int. J. Quantum Chem. 112, 2952 (2012). doi:10.1002/qua. 24140
[15] M.C. Heaven, J.M. Merritt and V.E. Bondybey, Annu. Rev. Phys. Chem. 62, 375 (2011). doi:10.1146/annurev-physchem-032210-102545
[16] M.M. Montero-Campillo, O. Mó, M. Yáñez, I. Alkorta, and J. Elguero, Adv. Inorg. Chem. 73, 73 (2019). doi:10.1016/bs.adioch.2018.10.003
[17] N.B. Jaffe, J.F. Stanton and M.C. Heaven, J. Phys. Chem. Lett. 14, 8339 (2023). doi:10.1021/acs.jpclett.3c02169
[18] A. Kalemos, J. Chem. Phys. 146, 104307 (2017). doi:10.1063/1.4977930
[19] A. Kalemos, J. Phys. Chem. A. 122, 8882 (2018). doi:10.1021/acs.jpca.8b07536
[20] A. Kalemos, Chem. Phys. Lett. 739, 136964 (2020). doi:10.1016/j.cplett.2019.136964
[21] A. Kramida, Yu. Ralchenko, J. Reader, and NIST ASD Team (2022). NIST Atomic Spectra Database (ver. 5.10), [Online]. Available: https://physics.nist.gov/asd. National Institute of Standards and Technology, Gaithersburg, MD.
[22] T.H.Jr. Dunning, J. Chem. Phys. 90, 1007 (1989). doi:10.1063/1.456153
[23] MOLPRO is a package of ab initio programs written by H.-J. Werner, P. J. Knowles, G. Knizia, F. R. Manby, M. Schütz, P. Celani, W. Györffy, D. Kats, T. Korona, R. Lindh, A. Mitrushenkov, G. Rauhut, K. R. Shamasundar, T. B. Adler, R. D. Amos, A. Bernhardsson, A. Berning, D. L. Cooper, M. J. O. Deegan, A. J. Dobbyn, F. Eckert, E. Goll, C. Hampel, A. Hesselmann, G. Hetzer, T. Hrenar, G. Jansen, C. Köppl, Y. Liu, A. W. Lloyd, R. A. Mata, A. J. May, S. J. McNicholas, W. Meyer, M. E. Mura, A. Nicklaß, D. P. O'Neill, P. Palmieri, D. Peng, K. Pflüger, R. Pitzer, M. Reiher, T. Shiozaki, H. Stoll, A. J. Stone, R. Tarroni, T. Thorsteinsson and M. Wang. MOLPRO, version 2012.1, a package of ab initio programs; University College Cardiff Consultants Limited: Cardiff, U.K., 2008.

