LETTERS TO THE EDITOR

Letters are selected for their expected interest for our readers. Some letters are sent to reviewers for advice; some are accepted or declined by the editor without review. Letters must be brief and may be edited, subject to the author's approval of significant changes. Although some comments on published articles and notes may be appropriate as letters, most such comments are reviewed according to a special procedure and appear, if accepted, in the Notes and Discussions section. (See the "Statement of Editorial Policy" in the January issue.) Running controversies among letter writers will not be published.

COMMENT ON "CONNECTING THERMODYNAMICS TO STUDENTS' CALCULUS," BY JOEL W. CANNON [AM. J. PHYS. 72 (6), 753–757 (2004)]

In a recent paper ["Connecting thermodynamics to students' calculus," Am. J. Phys. **72** (6), 753–757 (2004)], Joel W. Cannon makes some very good points about the usefulness of introducing Legendre transforms to students in order to clarify the difference between the functions and the independent variables in a system when calculating partial derivatives. I offer here an example with which students are usually very familiar, that is, the electrical power, P, of a resistor with resistance R when the potential difference across the terminals is V and the current is I. Recall that $P=IV=I^2R=V^2/R$. Depending on the variables held constant, that is, depending on which variables are independent, partial derivatives yield different results: $(\partial P/\partial I)_V=V$, whereas $(\partial P/\partial I)_R=2IR=2V$. Similarly, $(\partial P/\partial R)_I=I^2$, while $(\partial P/\partial R)_V=-V^2$ has the opposite sign. Derivatives provide the answer to questions such as "Does the power go up or down, if the resistance is increased?" These results show that the answer is not unique if all conditions are not explicitly stated (for example, constant current or constant voltage?), or, in other words, if it is not clarified which are the independent variables.

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PARALLEL UNIVERSES IN THE STATISTICS LITERATURE

York *et al.* recently considered straight-line regression when random errors are present in both the dependent and independent variables;[1] moreover, these errors may be correlated with each other. They compared the least-squares and maximum likelihood approaches for estimating the regression parameters and their standard errors, and found that the two approaches are equivalent when the least-squares approach uses properly adjusted data.

In the mathematical statistics community there is a great deal of literature on regression with errors in both the dependent and independent variables, including generalizations such as nonlinear curve fits, multiple predictor variables, and non-normal errors. In statistics, models of this type are called *measurement error models* or *errors-in-variables models*. For linear models of this type, the definitive monograph is by Fuller;[2] a more recent (and perhaps more accessible) monograph is by Cheng and Van Ness.[3] Nonlinear models are discussed by Carroll *et al.*[4] This literature seems to inhabit a "parallel universe" in that Ref. 1 and the papers they cite make no reference to these books, and *vice versa*. However, the two parallel sets of literature cite common origins, including the work of Adcock in the 1870s and that of Deming, a physicist turned statistician who wrote about the problem in the 1930s and 40s.

It might appear that the two parallel universes have diverged in terms of the problems they address, but there is a strong possibility that at least subtle connections exist between them. Such connections are probably obscured by what looks (at first glance) to be an excessively baroque theoretical apparatus that has emerged in the mathematical statistics literature. This baroqueness is due to the statisticians' goal of systematically addressing a vast array of general data analysis problems far beyond those that typically arise in the physical sciences. It does not help that Refs. 2–4 are addressed to professional statisticians rather than experimental scientists.

One can find comparisons of least-squares and maximum likelihood techniques in the mathematical statistics literature. For instance, in discussing a methodology called *modified least squares*, Ref. 3 states that "The beauty of modified least squares is that it is a unified approach; however, it does not introduce any new estimators that were not already available from maximum likelihood or the method of moments" (p. 89). The modified least squares approach is unified in that it can be applied to several different statistical models using various assumptions for the nature of the errors. Unfortunately, as a nonexpert I have been unable to determine the relation between this approach and that of Ref. 1. I surmise that, with effort, the specific problem of Ref. 1 could be formulated in the framework of mathematical statistics.

The authors of Ref. 1 deserve much praise for providing a direct, succinct solution to their problem which has not appeared elsewhere in its current form. Perhaps physicists with similar data analysis problems also will find the literature in mathematical statistics worthy of further exploration.

[1] D. York, N. M. Evensen, M. López Martínez, and J. De Basabe Delgado, "Unified equations for the slope, intercept, and standard errors of the best straight line," Am. J. Phys. **72**, 367–375 (2004).
[2] W. A. Fuller, *Measurement Error Models* (Wiley, New York, 1987).
[3] C.-L. Cheng and J. W. Van Ness, *Statistical Regression with Measurement Error* (Arnold, London, 1999).
[4] R. J. Carroll, D. Ruppert, and L. A. Stefanski, *Measurement Error in Nonlinear Models* (Chapman & Hall, London, 1995).

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